Bell inequalities with no quantum violation and unextendable product bases

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Abstract

The strength of classical correlations is subject to certain constraints, commonly known as Bell inequalities [1]. Violation of these inequalities is the manifestation of non-locality displayed in particular by quantum mechanics, meaning that quantum mechanics can outperform classical physics at tasks associated with such Bell inequalities. It is then an interesting question as to whether there are nontrivial tasks (the associated Bell inequality is tight, i.e., it defines the facet of the polytope of classical correlations) at which quantum correlations are at most as powerful as classical correlations but still the corresponding Bell inequality is nontrivial and can be violation by some nonsignalling correlations. Quite recently this question was answered affirmatively in the case when the number of observers is greater than two and examples of such tasks (Bell inequalities) were proposed [2].

With this work [3] we relate these Bell inequalities with another interesting object known rather in the theory of entanglement, i.e., unextendable product basis (UPB) [4]. The latter is a collection of product vectors $\mathcal{S} = \{|\phi_j^{(1)}\rangle \otimes \ldots \otimes |\phi_j^{(N)}\rangle\}_{j=1}^{|\mathcal{S}|}$ from a multipartite Hilbert space $\mathcal{H} = \mathbb{C}^{d_1} \otimes \ldots \otimes \mathbb{C}^{d_N}$ such that they span a proper subspace in H and there is no product vector in H orthogonal to $\mathcal{S}$. The importance of UPBs in quantum information theory stems for instance from the fact that they allow for a simple construction [4] of bound entangled states, i.e., states that are entangled but from which no entanglement can be distilled by means of local operations and classical communication [5].

We show [3] that any UPB possessing certain property can be associated with a Bell inequality that is not violated by quantum states, meaning that it represents a distributed task at which quantum correlations perform at most as well as the classical ones. Nevertheless, there still exist supraquantum nonsignalling correlations (correlations that cannot be achieved by measuring quantum states, but still are compatible with special relativity) that violate this Bell inequality, i.e., they provide an advantage over the classical and quantum ones at the task corresponding to this Bell inequality. Finally, we study properties of the Bell inequalities obtained from UPB living in the simplest multipartite Hilbert space $(\mathbb{C}^2)^{\otimes N}$ and find new examples of UPBs leading to tight Bell inequalities with no quantum violation [6].

References

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