Frustration and time reversal symmetry breaking for Fermi and Bose-Fermi systems

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Complex tunneling amplitudes

A. Eckardt, C. Weiss, and M. Holthaus, PRL **95**, 260404 (2005).
J. Struck *et al.*, Science **333**, 996 (2011).

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Tunable Gauge Potential for Neutral and Spinless Particles in Driven Optical Lattices

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Time-revesal Symmetry and Quantum Chaos

KS, J. Zakrzewski and D. Delande, PRL, **83**, 2922 (1999),

KS, and J. Zakrzewski, PRL **86**, 2269 (2001).

$$H_0 = \frac{p^2}{2m} + V(x) + K_1 x \cos(\omega t) + K_2 x \cos(2\omega t + \varphi), \quad t > 0$$

$$\mathcal{H} = H_0 - i\hbar \frac{\partial}{\partial t}, \qquad \mathcal{H} \left| u_n(t) \right\rangle = E_n \left| u_n(t) \right\rangle$$

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$$\mathcal{H} = H_0 - i\hbar\frac{\partial}{\partial t}, \qquad \mathcal{H}\,|u_n(t)\rangle = E_n\,|u_n(t)\rangle$$

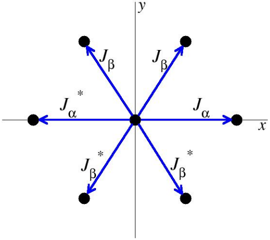
$$\phi_{j,m}(x,t) = e^{-ix\left[\frac{K_1}{\omega}\sin(\omega t)+\frac{K_2}{2\omega}\sin(2\omega t+\varphi)\right]}\; e^{im\omega t}\; W_j(x),$$

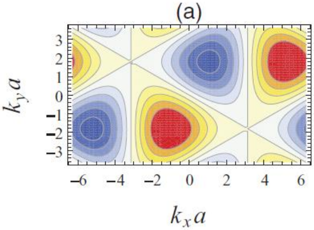
$$\langle\phi_{j',0}|\mathcal{H}|\phi_{j,0}\rangle = -J_{\rm eff}\;\delta_{j',j+1} - J^*_{\rm eff}\;\delta_{j',j-1}$$

$$J_{\rm eff} = J\sum_{k=-\infty}^{\infty}\mathcal{J}_{2k}\left(\frac{aK_1}{\omega}\right)\mathcal{J}_k\left(\frac{aK_2}{\omega}\right)e^{ik\varphi},$$

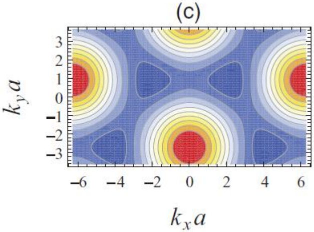
2D triangular lattice

$$J_\alpha = |J_\alpha|\ e^{i\varphi_\alpha}, \qquad J_\beta = |J_\beta|\ e^{i\varphi_\beta}$$





$$\varphi_\alpha = \varphi_\beta = \frac{\pi}{2}$$
$$|J_\alpha| = |J_\beta|$$



$$\varphi_\alpha = \pi$$
$$\varphi_\beta = \frac{\pi}{4}$$
$$|J_\alpha| = |J_\beta|$$

Superfuid Fermi mixture in a triangular lattice

BCS approach

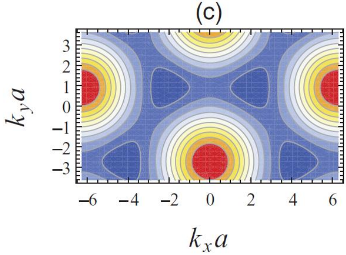
$$
\begin{aligned}
H_{F,\mathrm{eff}} &= -\sum_{\langle ij \rangle} J_{ij}\left(\hat{a}^{\dagger}_{i\uparrow}\hat{a}_{j\uparrow} + \hat{a}^{\dagger}_{i\downarrow}\hat{a}_{j\downarrow}\right) - \mu \sum_{i}\left(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow}\right) \\
&\quad + \sum_{i}\left(\Delta_i\ \hat{a}^{\dagger}_{i\uparrow}\hat{a}^{\dagger}_{i\downarrow} + \Delta_i^{*}\ \hat{a}_{i\downarrow}\hat{a}_{i\uparrow}\right)
\end{aligned}
$$

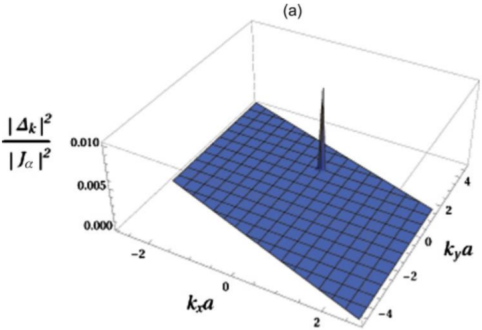
$$
\begin{bmatrix} u_{\mathbf{k}}(\mathbf{r}_i) \\ v_{\mathbf{k}}(\mathbf{r}_i) \end{bmatrix} \propto \begin{bmatrix} U_{\mathbf{k}}\ e^{i\mathbf{k}_0\cdot\mathbf{r}_i} \\ V_{\mathbf{k}}\ e^{-i\mathbf{k}_0\cdot\mathbf{r}_i} \end{bmatrix} e^{i\mathbf{k}\cdot\mathbf{r}_i}
$$

$$
\Delta_i = U \sum_{\mathbf{k}} u_{\mathbf{k}}(\mathbf{r}_i)\ v^{*}_{\mathbf{k}}(\mathbf{r}_i)\ \left[1 - 2\theta(-\varepsilon_{\mathbf{k}})\right] = e^{i2\mathbf{k}_0\cdot\mathbf{r}_i}\ |\Delta|.
$$

Superfuid Fermi mixture in a triangular lattice

$$\varphi_\alpha = \pi, \quad \varphi_\beta = \frac{\pi}{4}, \quad |J_\alpha| = |J_\beta|, \quad \frac{U}{|J_\alpha|} = 2, \quad \mu = 0$$





$$\Delta_{\mathbf{k}} = \frac{1}{\sqrt{N_s}} \sum_i \Delta_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}$$

60 × 60 lattice sites

Bose-Fermi mixture in a triangular lattice

$$
\begin{aligned}
H_{F,\mathrm{eff}} \quad = \quad & -\sum_{\langle ij \rangle} J_{ij} \left(\hat{a}^\dagger_{i\uparrow} \hat{a}_{j\uparrow} + \hat{a}^\dagger_{i\downarrow} \hat{a}_{j\downarrow} \right) - \mu \sum_i \left(\hat{n}_{i\uparrow} + \hat{n}_{i\downarrow} \right) \\
& + \sum_i \left(\Delta_i \, \hat{a}^\dagger_{i\uparrow} \hat{a}^\dagger_{i\downarrow} + \Delta^*_i \, \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} \right) + \gamma \sum_i \left(\psi^*_i \, \hat{a}_{i\downarrow} \hat{a}_{i\uparrow} + \psi_i \, \hat{a}^\dagger_{i\uparrow} \hat{a}^\dagger_{i\downarrow} \right)
\end{aligned}
$$

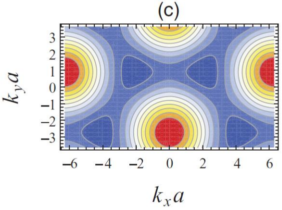
where BEC wavefunction $\psi_i = \sqrt{n_B} \, e^{i\mathbf{q}_0 \cdot \mathbf{r}_i}$

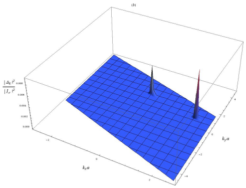
$$
\begin{bmatrix} u_{\mathbf{k}}(\mathbf{r}_i) \\ v_{\mathbf{k}}(\mathbf{r}_i) \end{bmatrix} \propto \begin{bmatrix} U_{\mathbf{k}} \; e^{i\mathbf{k}_0 \cdot \mathbf{r}_i} \\ V_{\mathbf{k}} \; e^{-i\mathbf{k}_0 \cdot \mathbf{r}_i} \end{bmatrix} e^{i\mathbf{k} \cdot \mathbf{r}_i} = \begin{bmatrix} U_{\mathbf{k}} \; e^{i\mathbf{q}_0 \cdot \mathbf{r}_i / 2} \\ V_{\mathbf{k}} \; e^{-i\mathbf{q}_0 \cdot \mathbf{r}_i / 2} \end{bmatrix} e^{i\mathbf{k} \cdot \mathbf{r}_i}
$$

$$
\Delta_i = \; e^{i\mathbf{q}_0 \cdot \mathbf{r}_i} \, |\Delta|.
$$

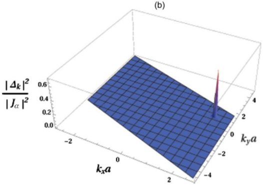
Bose-Fermi mixture in a triangular lattice

$$\varphi_\alpha = \pi, \quad \varphi_\beta = \frac{\pi}{4}, \quad |J_\alpha| = |J_\beta|, \quad \frac{U}{|J_\alpha|} = 2, \quad \mu = 0$$





$$\frac{\gamma\sqrt{n_B}}{|J_\alpha|} = 0.2$$



$$\frac{\gamma\sqrt{n_B}}{|J_\alpha|} = 2.3$$

- An increase of γ causes a gradual reduction of the peak at $\mathbf{k} = \left(0, \frac{\pi}{a\sqrt{3}}\right)$ together with an enlargement of the peak at $\mathbf{k} = \mathbf{q}_0$.
- At $\gamma \approx 0.3 \frac{|J_\alpha|}{\sqrt{n_B}}$ there is a crossover where the ground state starts to be well reproduced by the anlytical solution.
- For $\gamma < 2.1 \frac{|J_\alpha|}{\sqrt{n_B}}$ anlytical solution describes gapless superfluidity (no gap in the excitation spectrum but $\Delta \neq 0$).

$$\Delta_{\mathbf{k}} = \frac{1}{\sqrt{N_s}} \sum_i \Delta_i e^{-i\mathbf{k}\cdot\mathbf{r}_i}$$

60×60 lattice sites

Summary

- Complex tunneling amplitudes in a lattice system can be realized by means of a periodic shaking that breaks time-reversal symmetry.
- Ground state of a superfluid Fermi system with broken time-reversal symmetry can be described by the pairing function with a non-trivial quasi-momentum.
- If bosons are also present in the lattice and there is sufficiently strong coupling between fermions and bosons, the phase of the BCS pairing function reflects the phase of the BEC wave function.
- In the presence of bosons the ground state of the Fermi system can reveal both gapped and gapless superfluidity.

KS, K. Targońska, J. Zakrzewski, PRA 85, 053613 (2012).