An impurity in a Fermi sea on a narrow Feshbach resonance: A variational study of the polaronic and dimeronic branches

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dépasser les frontières

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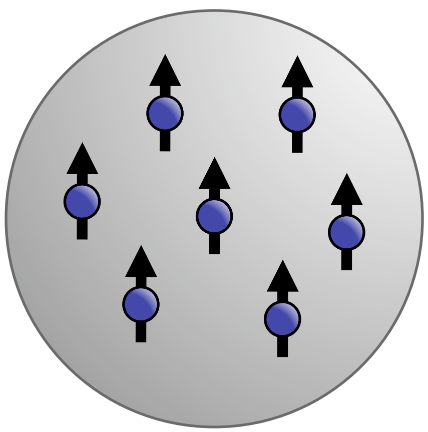
MARIE CURIE ACTIONS

Introduction: The system

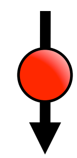
Homogeneous (3D) Fermi gas of same spin state

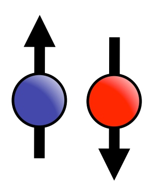
m, k_{F}



Distinguishable impurity (boson/fermion)

M

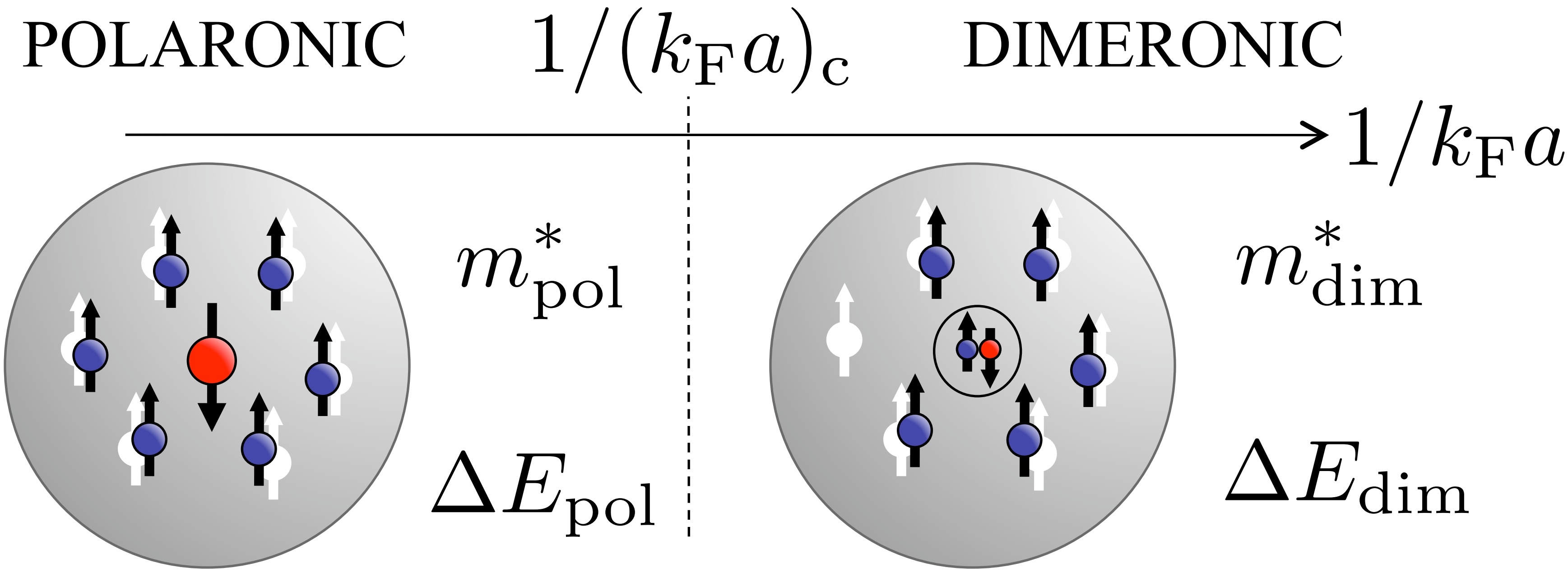


s-wave interactions  : Scattering length a

Magnetic Feshbach resonance: Change a at will !!

Introduction: The system

The ground state has two quasiparticle branches:



C. Lobo *et. al.*, PRL **97**, 200403 (2006)

F. Chevy, PRA **74**, 063628 (2006)

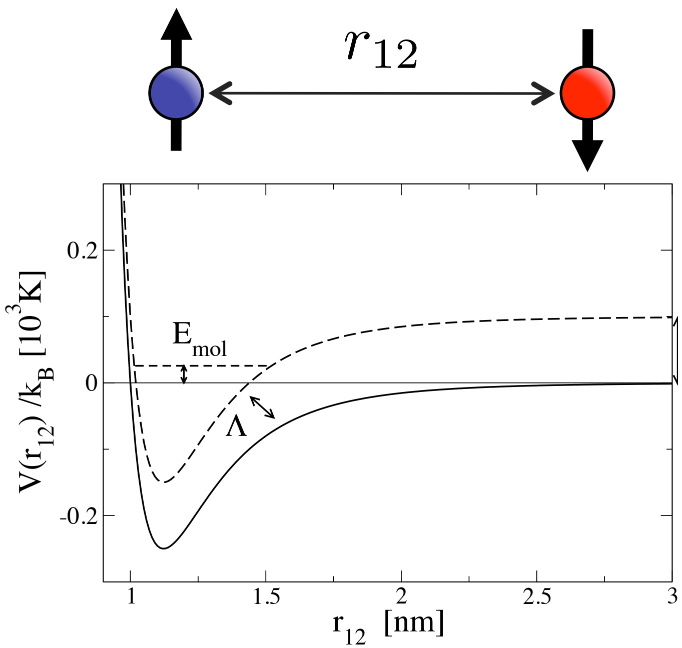
N. Prokof'ev *et. al.*, PRB **77**, 020408 (2008)

M. Punk *et. al.*, PRA **80**, 053605 (2009)

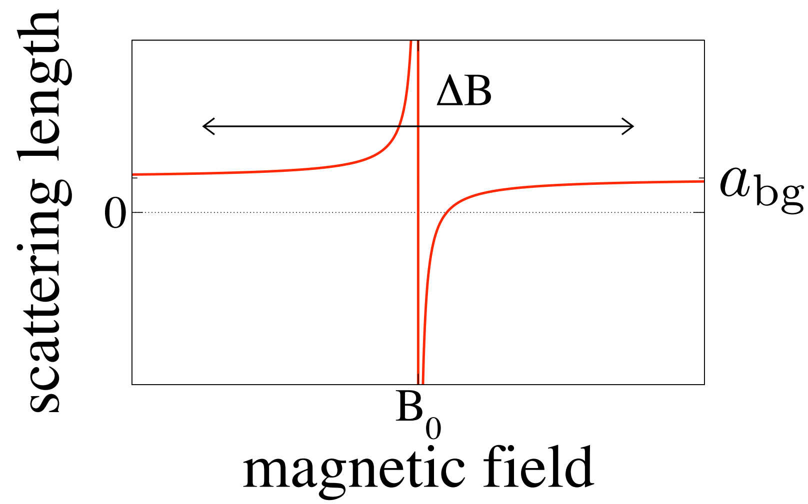
C. Mora, F. Chevy, PRA **80**, 033607 (2009)

R. Combescot *et. al.*, EPL **88**, 60007 (2009)

Introduction: Narrow Feshbach resonance



$$a = a_{\rm bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$



$$R_* = \frac{\pi\hbar^4}{\Lambda^2\mu^2} \propto \frac{1}{\Delta B}$$ FESHBACH LENGTH

$\Delta B \equiv$ RESONANCE WIDTH

(i) Theoretically: Extra parameter R_* effect on the polaron/dimeron ?

(ii) Experimentally: $^{40}\mathrm{K} - ^{6}\mathrm{Li}$ mixtures, narrow Feshbach resonances

$$R_* > 100\mathrm{nm} \gg R_{\rm VdW} \simeq 2\mathrm{nm}$$

Outlook

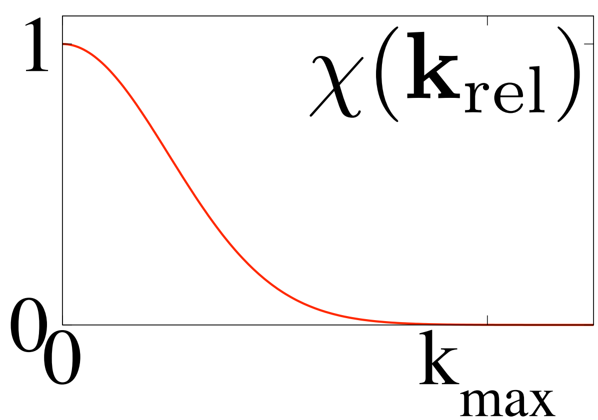
1. Two-channel model Hamiltonian
2. Variational ansatz
3. Integral equations: Polaron, dimeron
4. A discrete state coupled to a continuum
5. Properties of the two branches at **P**=**0**:
   a) Polaron-to-dimeron crossing point
6. Non-trivial weakly interacting limit

Two-channel model Hamiltonian

$$\hat{H} = \sum_{\mathbf{k}} \left[\varepsilon_{\mathbf{k}} \hat{u}^\dagger_{\mathbf{k}} \hat{u}_{\mathbf{k}} + E_{\mathbf{k}} \hat{d}^\dagger_{\mathbf{k}} \hat{d}_{\mathbf{k}} + \left(\frac{\varepsilon_{\mathbf{k}}}{1+r} + E_{\mathrm{mol}} \right) \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}} \right] + \frac{\Lambda}{\sqrt{V}} \sum_{\mathbf{k},\mathbf{k}'} \chi(\mathbf{k}_{\mathrm{rel}}) (\hat{b}^\dagger_{\mathbf{k}+\mathbf{k}'} \hat{u}_{\mathbf{k}} \hat{d}_{\mathbf{k}'} + \mathrm{h.c.})$$

$$\varepsilon_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m}, \quad E_{\mathbf{k}} = \frac{\hbar^2 k^2}{2M}, \quad r = \frac{M}{m}, \quad \mathbf{k}_{\mathrm{rel}} = \mu \left(\frac{\mathbf{k}}{m} - \frac{\mathbf{k}'}{M} \right) \quad \mu = \frac{mM}{m+M}$$

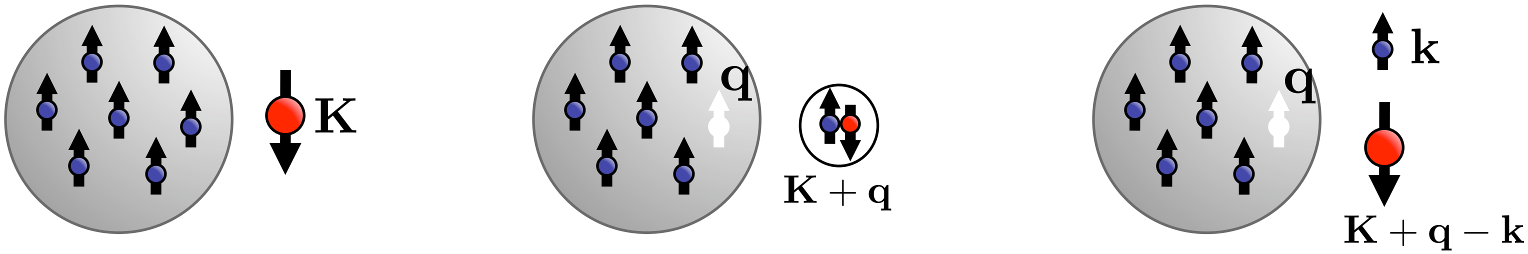
$$\{\hat{u}_{\mathbf{k}}, \hat{u}^\dagger_{\mathbf{k}'}\} = \delta_{\mathbf{k}\mathbf{k}'}, \quad \{\hat{d}_{\mathbf{k}}, \hat{d}^\dagger_{\mathbf{k}'}\} = \delta_{\mathbf{k}\mathbf{k}'}, \quad [\hat{b}_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k}\mathbf{k}'}$$



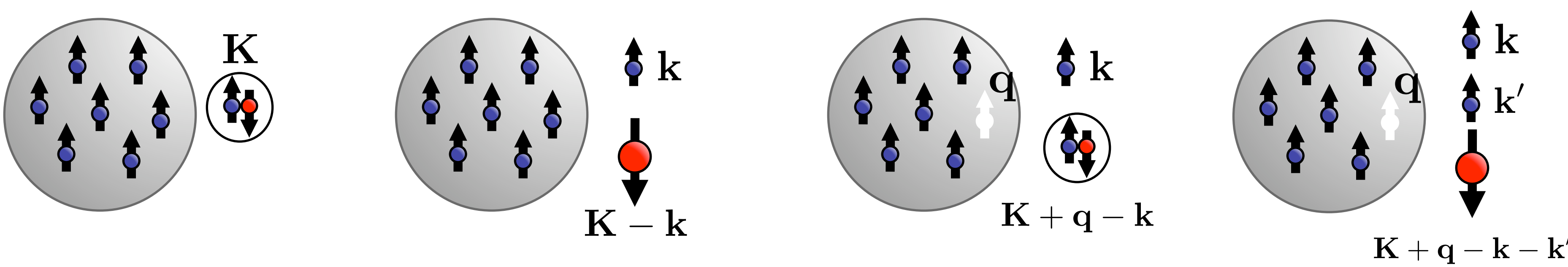
$$\lim_{k_{\max} \to +\infty} \chi \to 1$$

Variational ansatz, total momentum $\mathbf{P} = \hbar \mathbf{K}$

$$|\psi_{\rm pol}(\mathbf{P})\rangle = \Big(\phi\, \hat{d}^\dagger_{\mathbf{K}} + {\sum_{\mathbf{q}}}' \phi_{\mathbf{q}} \hat{b}^\dagger_{\mathbf{K}+\mathbf{q}} \hat{u}_{\mathbf{q}} + {\sum_{\mathbf{k},\mathbf{q}}}' \phi_{\mathbf{k}\mathbf{q}} \hat{d}^\dagger_{\mathbf{K}+\mathbf{q}-\mathbf{k}} \hat{u}^\dagger_{\mathbf{k}} \hat{u}_{\mathbf{q}} \Big) |\mathrm{FS}:N\rangle$$



$$|\psi_{\rm dim}(\mathbf{P})\rangle = \Big(\eta\, \hat{b}^\dagger_{\mathbf{K}} + {\sum_{\mathbf{k}}}' \eta_{\mathbf{k}} \hat{d}^\dagger_{\mathbf{K}-\mathbf{k}} \hat{u}^\dagger_{\mathbf{k}} + {\sum_{\mathbf{k},\mathbf{q}}}' \eta_{\mathbf{k}\mathbf{q}} \hat{b}^\dagger_{\mathbf{K}+\mathbf{q}-\mathbf{k}} \hat{u}^\dagger_{\mathbf{k}} \hat{u}_{\mathbf{q}} + {\sum_{\mathbf{k}',\mathbf{k},\mathbf{q}}}' \eta_{\mathbf{k}'\mathbf{k}\mathbf{q}} \hat{d}^\dagger_{\mathbf{K}+\mathbf{q}-\mathbf{k}-\mathbf{k}'} \hat{u}^\dagger_{\mathbf{k}'} \hat{u}^\dagger_{\mathbf{k}} \hat{u}_{\mathbf{q}} \Big) |\mathrm{FS}:N-1\rangle$$



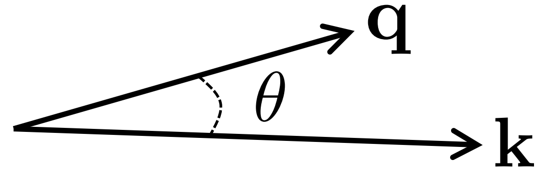
Integral equations in the thermodynamic limit

$$\Delta E_{\mathrm{pol}}(\mathbf{P}) = E_{\mathbf{K}} + \int' \frac{d^3 q}{(2\pi)^3} \frac{1}{D_{\mathbf{q}}\left[\Delta E_{\mathrm{pol}}(\mathbf{P}), \mathbf{P}\right]}$$

$$D_{\mathbf{q}}(E, \mathbf{P}) = \frac{1}{g} - \frac{\mu k_{\mathrm{F}}}{\pi^2 \hbar^2} + \frac{\mu^2 R_*}{\pi \hbar^4} \left(E + \varepsilon_{\mathbf{q}} - \frac{\varepsilon_{\mathbf{K}+\mathbf{q}}}{1+r} \right)$$
$$+ \int' \frac{d^3 k'}{(2\pi)^3} \left(\frac{1}{E_{\mathbf{K}+\mathbf{q}-\mathbf{k}'} + \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{q}} - E} - \frac{2\mu}{\hbar^2 k'^2} \right)$$

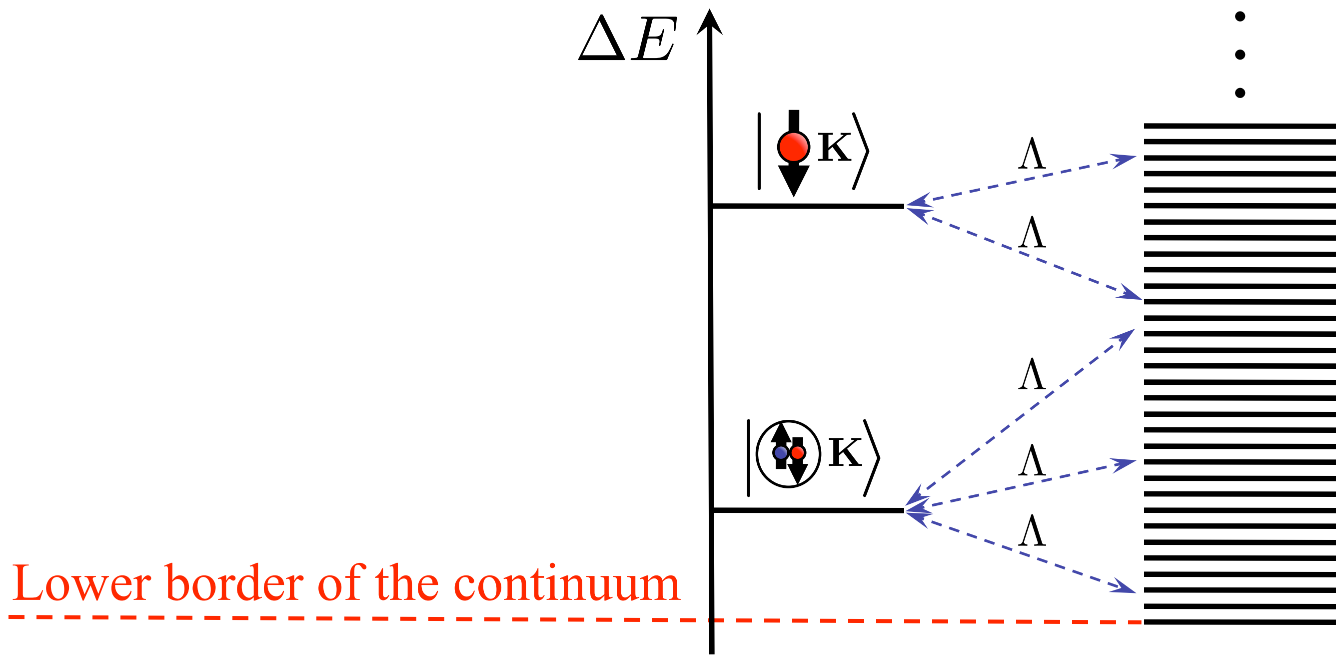
$$\int' \frac{d^3 k' d^3 q'}{(2\pi)^6} \mathcal{M}[\Delta E_{\mathrm{dim}}(\mathbf{P}), \mathbf{P}; \mathbf{k}, \mathbf{q}; \mathbf{k}', \mathbf{q}'] \eta_{\mathbf{k}'\mathbf{q}'} = 0$$

$$\eta_{\mathbf{kq}} = \eta(k, q, \theta)$$



Total of three parameters: $a, R_*, M/m$

A discrete state coupled to a continuum

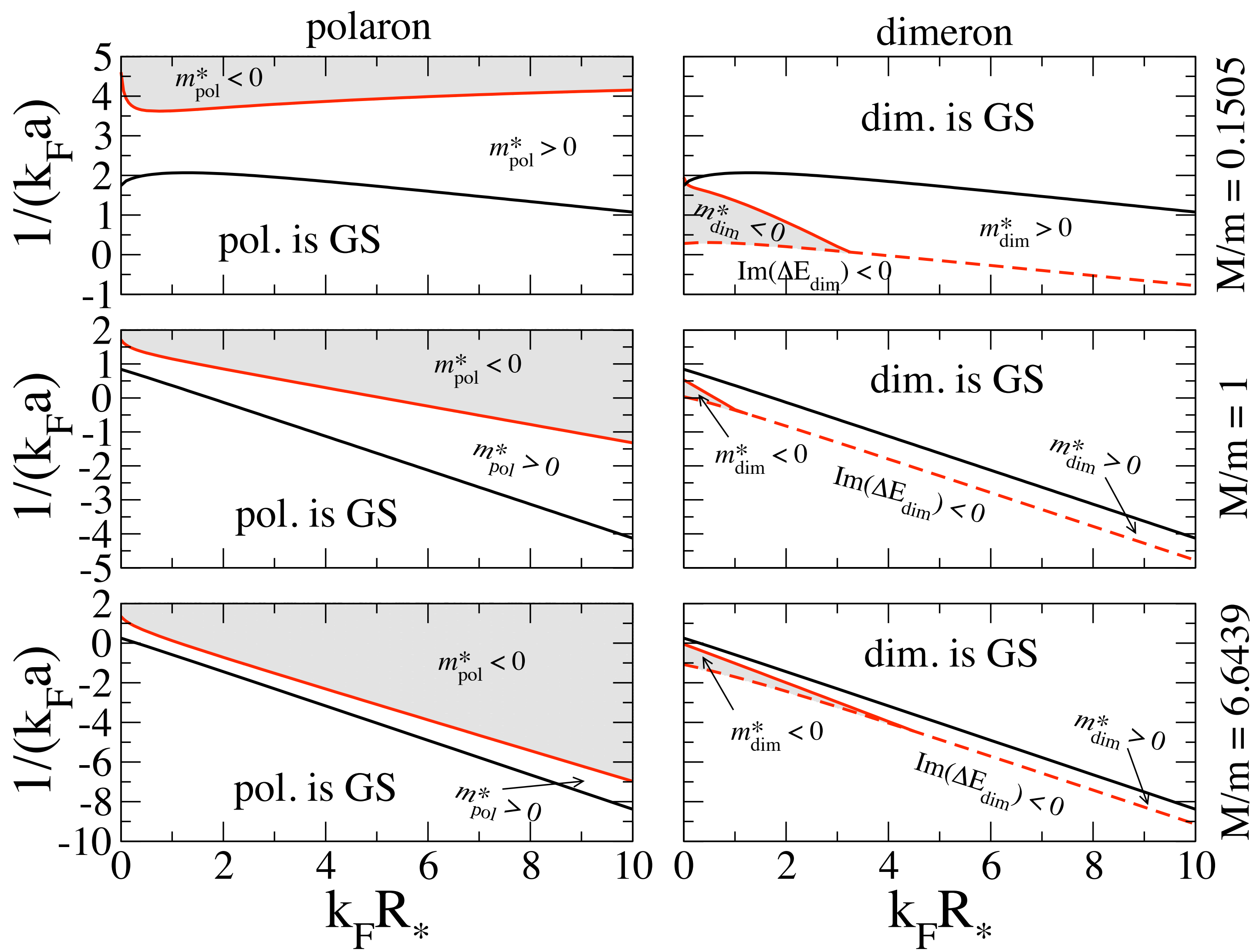


(1) Discrete state EXPELLED from the continuum, remains a discrete state.

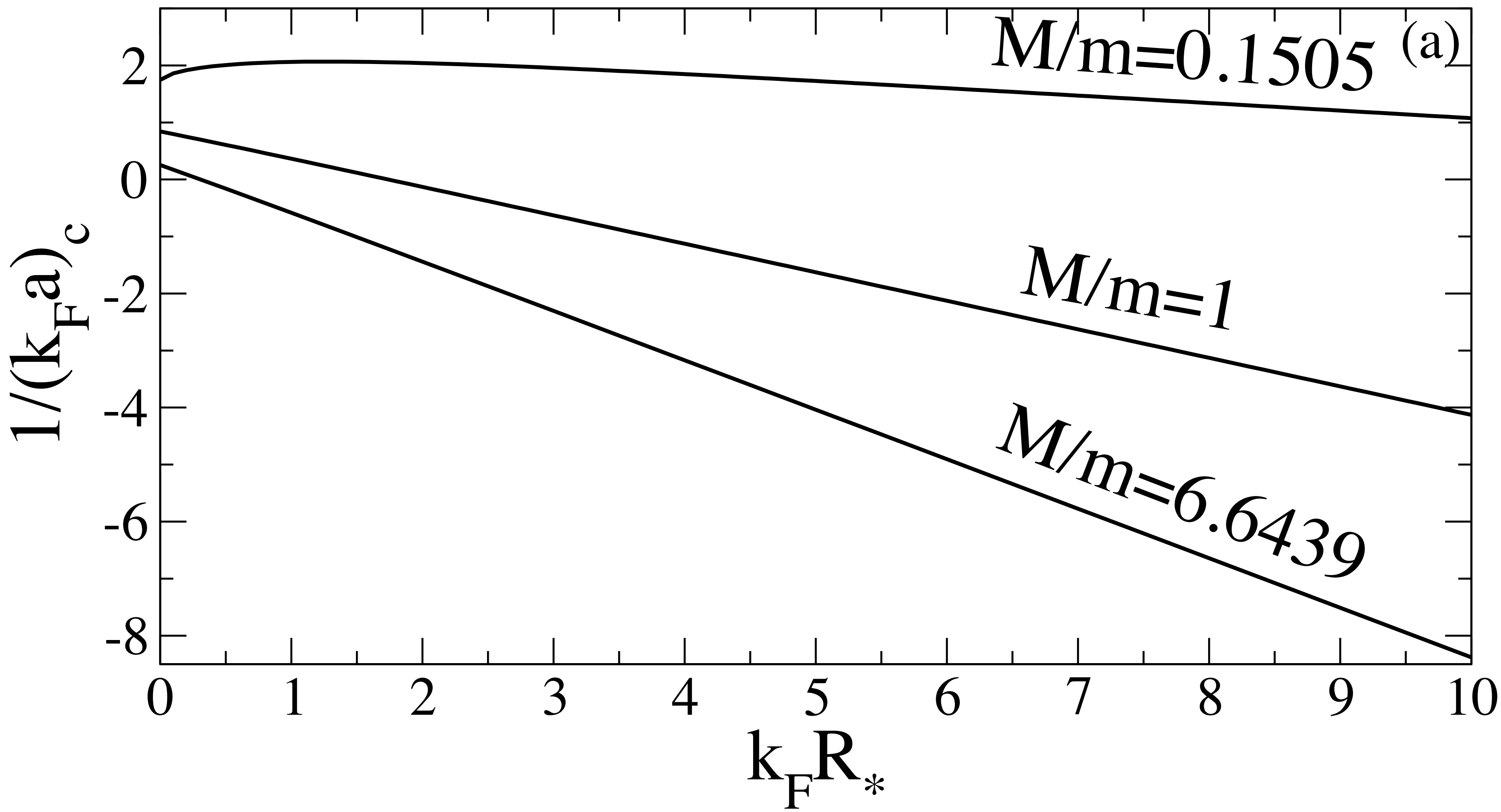
(2) Discrete state DILUTED in the continuum, becomes a resonance:

$$\Delta E \rightarrow \Delta E_R + i\Delta E_I \quad \text{of physical interest if} \quad \Delta E_I \ll \Delta E_R$$

Properties of the two branches at **P=0**



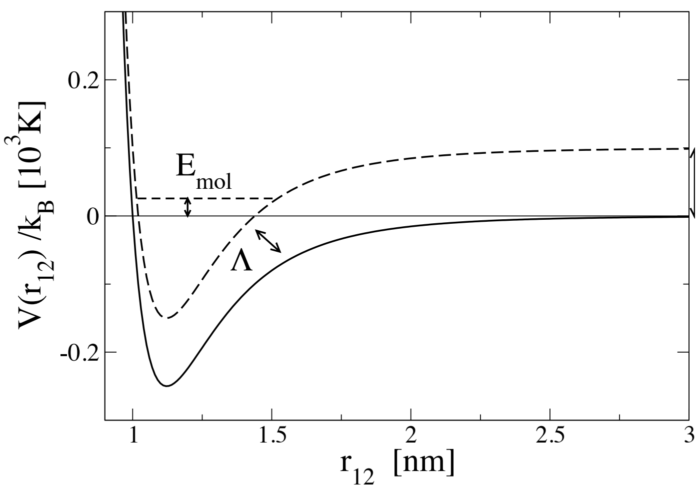
Polaron-to-dimeron crossing point



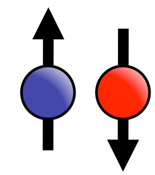
In vacuum, for ANY value of R_*, one has a two-body bound state iff $a > 0$

Intuitive picture

Question: When do we have a two-body bound state ?



(1) Two particles in vacuum:



(i) Naïve answer: $E_{\mathrm{mol}} < 0$

(ii) Lamb shift due to vacuum fluctuations in the open channel:

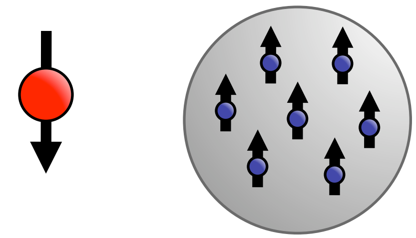
$$\widetilde{E}_{\mathrm{mol}} = E_{\mathrm{mol}} - \int \frac{d^3k}{(2\pi^3)} \frac{\chi^2(\mathbf{k})\Lambda^2}{\frac{\hbar^2k^2}{2\mu}} \Rightarrow \widetilde{E}_{\mathrm{mol}} = -\frac{\Lambda^2}{g}$$

$$\boxed{\widetilde{E}_{\mathrm{mol}} < 0 \Longleftrightarrow a > 0}$$

Intuitive picture

Question: When do we have a two-body bound state ?

(2) One impurity in a Fermi sea:



TWO EFFECTS

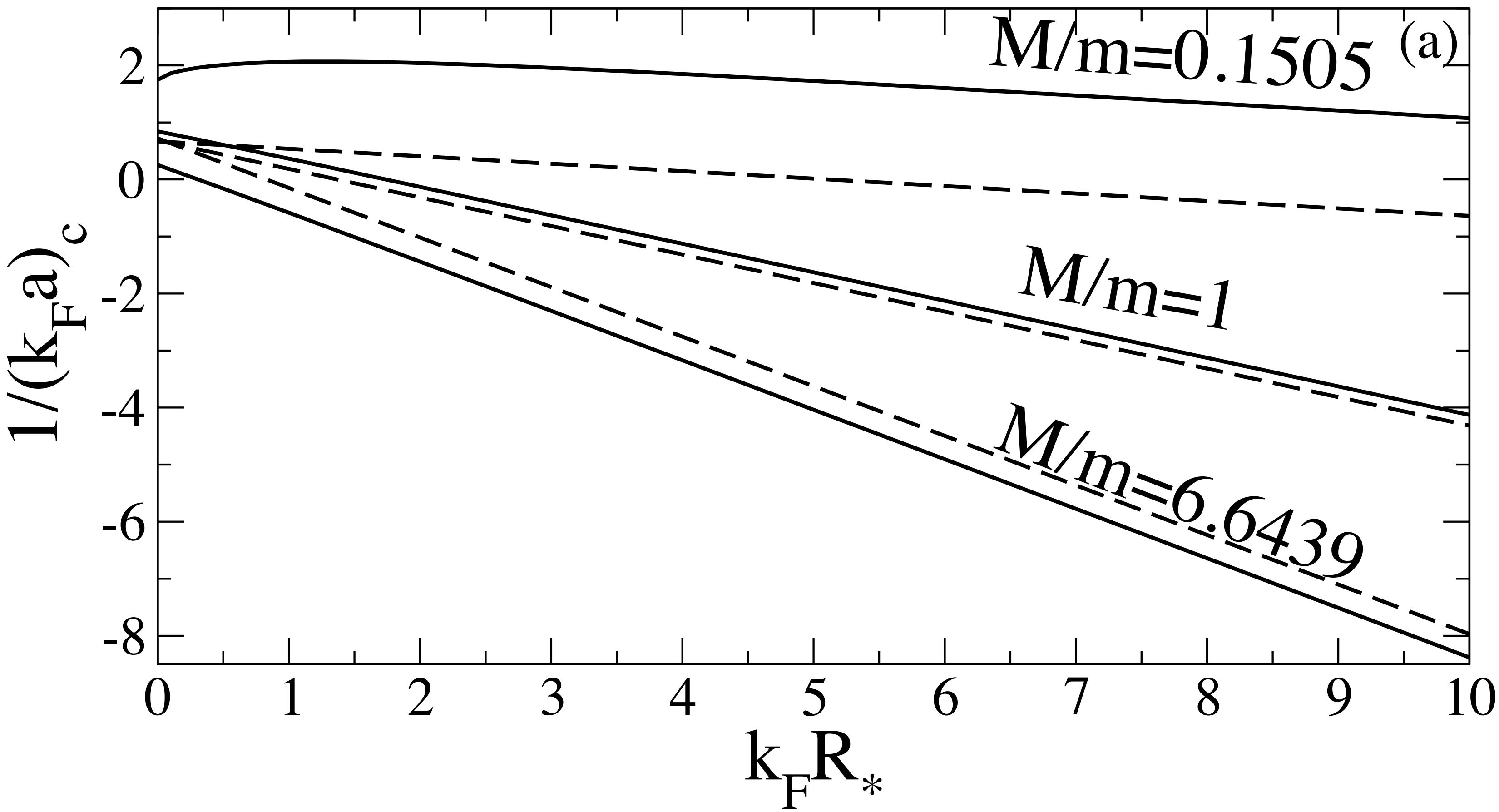
(i) Effect on the Lamb shift

$$\widetilde{E}'_{\mathrm{mol}} = E_{\mathrm{mol}} - \int_{k>k_{\mathrm{F}}} \frac{d^3k}{(2\pi)^3} \frac{\chi(\mathbf{k})\Lambda^2}{\frac{\hbar^2 k^2}{2\mu}}$$

(ii) Change of the dissociation threshold, E_{F}

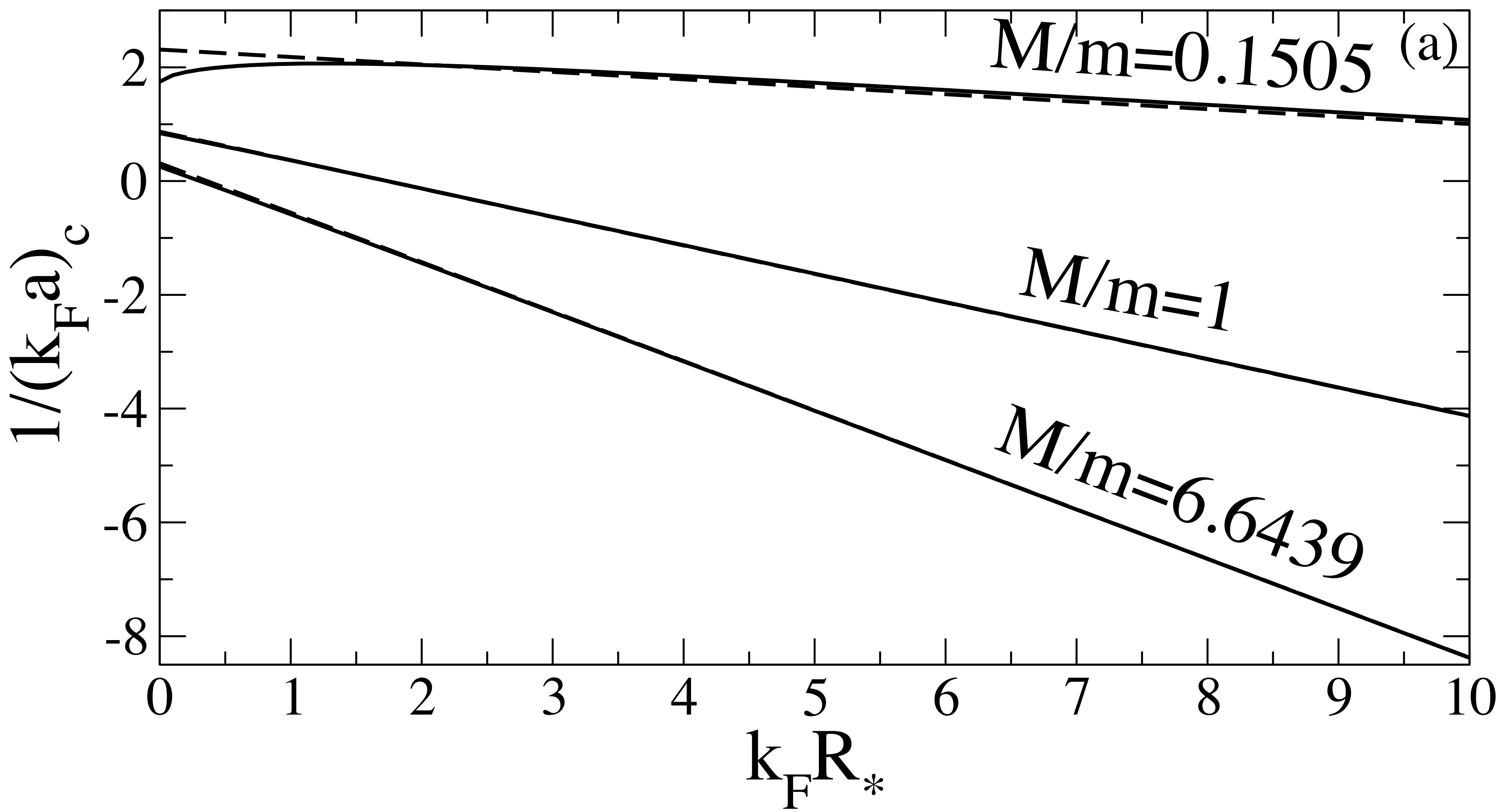
$$\widetilde{E}'_{\mathrm{mol}} < E_{\mathrm{F}} \Longrightarrow \quad \frac{1}{k_{\mathrm{F}} a} > \frac{2}{\pi} - \frac{M}{m+M} k_{\mathrm{F}} R_*$$

Test of intuitive picture



$$\frac{1}{k_{\rm F}a} = \frac{2}{\pi} - \frac{M}{m+M} k_{\rm F} R_*$$

Quantitative analytical result described later



Same slope as intuitive picture, but exact calculation of the INTERCEPT of the asymptote

Non-trivial weakly interacting limit

Standard weakly interacting limit:

$$a \rightarrow 0^-, R_* \text{ fixed!}$$

Loose information on the narrowness of the resonance

$$f_{\mathbf{k}} = \frac{-1}{\frac{1}{a} + ik + k^2 R_*}$$

Non-trivial weakly interacting limit:

$$a \rightarrow 0^-, \frac{1}{a} \text{ and } R_* \text{ proportional}$$

Non-trivial weakly interacting limit

FIXED

LIMIT: $\begin{cases} a \to 0^- \\ R_* \to +\infty \end{cases}$ ⇒ $s \equiv \frac{r}{1+r}(-aR_*)^{1/2}k_{\mathrm{F}}$

(1) Polaron: $\mathbf{P} = \mathbf{0}$

$$\Delta E_{\mathrm{pol}} = g \int' \frac{d^3q}{(2\pi)^3} \frac{1}{gD_{\mathbf{q}}\left[\Delta E_{\mathrm{pol}}, \mathbf{0}\right]} \qquad g = \frac{2\pi\hbar^2 a}{\mu}$$

⇒ $$gD_{\mathbf{q}}(\Delta E_{\mathrm{pol}}) \underset{a\to 0^-}{\longrightarrow} 1 - (sq/k_{\mathrm{F}})^2$$

⇒ $$\Delta E_{\mathrm{pol}} \underset{a\to 0^-}{\sim} 2E_{\mathrm{F}} \frac{1+r}{r} \frac{k_{\mathrm{F}}a}{\pi} \frac{1}{s^2} \left[\frac{1}{2s} \ln \frac{1+s}{1-s} - 1\right]$$

Non-trivial weakly interacting limit

FIXED

LIMIT: $\begin{cases} a \to 0^- \\ R_* \to +\infty \end{cases}$ ⇨ $s \equiv \frac{r}{1+r}(-aR_*)^{1/2}k_\mathrm{F}$

(2) Dimeron: $\mathbf{P} = \mathbf{0}$

$$D_\mathbf{0}(\Delta E_\mathrm{Cooper} + E_\mathrm{F}, \mathbf{0}) = 0 \qquad g = \frac{2\pi\hbar^2 a}{\mu}$$

⇨ $$\Delta E_\mathrm{Cooper} \underset{a\to 0^-}{\to} \Delta E^{(0)}_\mathrm{Cooper} \equiv E_\mathrm{F}\left[\frac{r}{1+r}\frac{1}{s^2} - 1\right]$$

⇨ $$\Delta E^{(1)}_\mathrm{Cooper} = -2E_\mathrm{F}\frac{r}{1+r}\frac{k_\mathrm{F}a}{\pi}\frac{1}{s^2}\left[1 - \frac{r}{2(1+r)s}\ln\frac{s\frac{1+r}{r}+1}{s\frac{1+r}{r}-1}\right]$$

Non-trivial weakly interacting limit

LIMIT: $\begin{cases} a \longrightarrow 0^- \\ R_* \longrightarrow +\infty \end{cases}$ ⇒ FIXED $\boxed{s \equiv \frac{r}{1+r}(-aR_*)^{1/2}k_{\mathrm{F}}}$

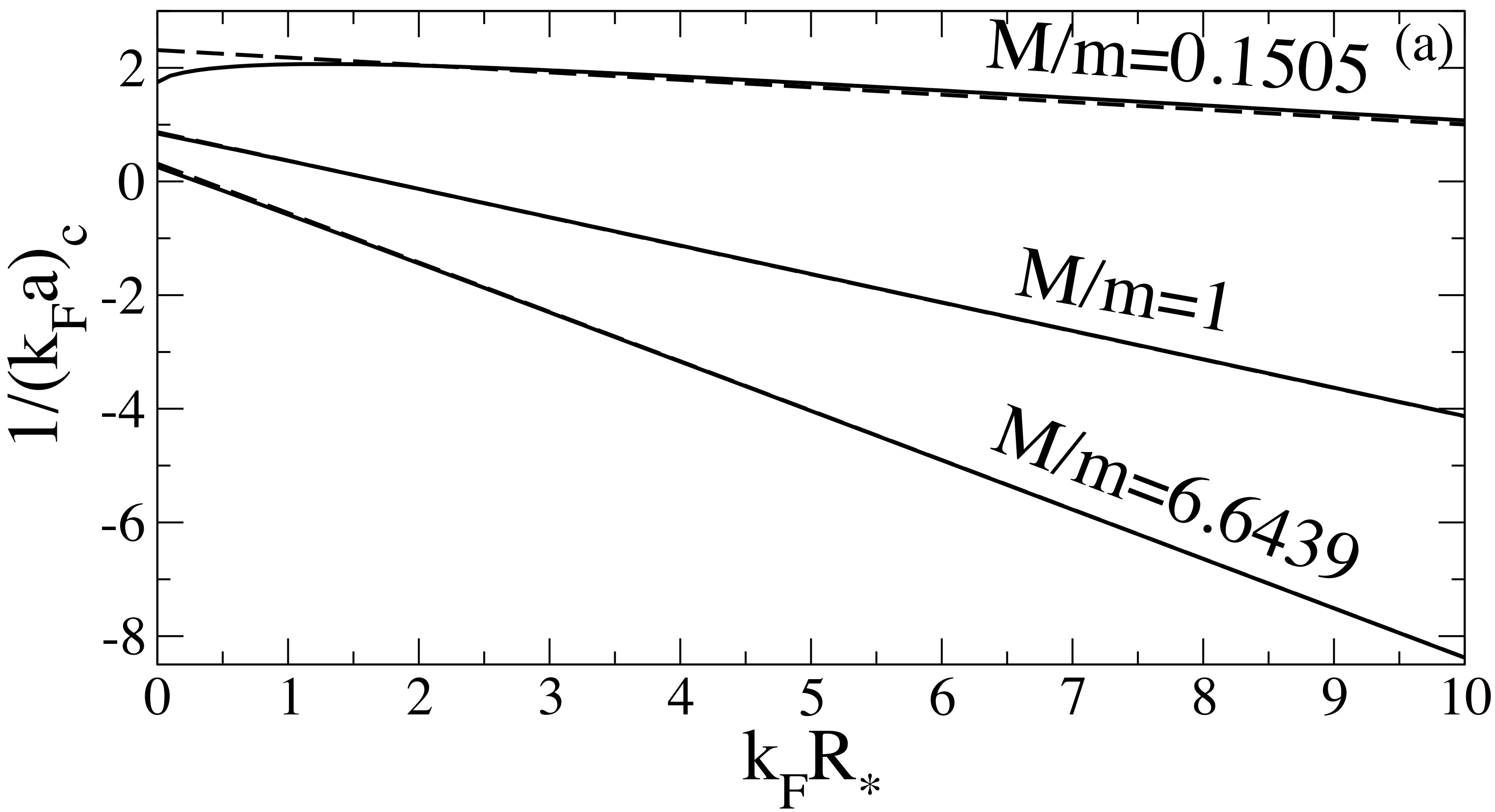
(3) Crossing: $\mathbf{P} = \mathbf{0}$

$$\Delta E^{(0)}_{\mathrm{Cooper}} + \Delta E^{(1)}_{\mathrm{Cooper}} = \Delta E_{\mathrm{pol}}$$

$$\left(\frac{1}{k_{\mathrm{F}}a}\right)_c \underset{R_*\to+\infty}{=} -\frac{r}{1+r}k_{\mathrm{F}}R_*$$

$$+\frac{2}{\pi}\left\{1-\left(\frac{1+r}{r}\right)^2+\frac{1}{2}\left[\left(\frac{1+r}{r}\right)^{5/2}-\left(\frac{r}{1+r}\right)^{1/2}\right]\ln\frac{1+\left(\frac{r}{1+r}\right)^{1/2}}{1-\left(\frac{r}{1+r}\right)^{1/2}}\right\}$$

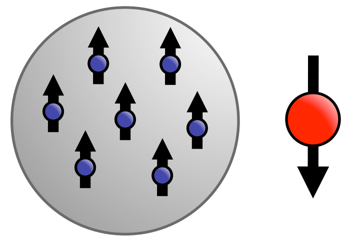
Non-trivial weakly interacting limit

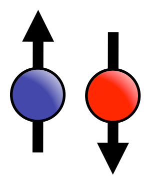


$M/m = 1,\ R_* = 0 :$ analytics $\neq$ numerics $\rightarrow$ 3%

Conclusions

Homogeneous (3D) Fermi gas of same spin state



s-wave interactions  : Narrow Feshbach resonance

A new lengthscale appears

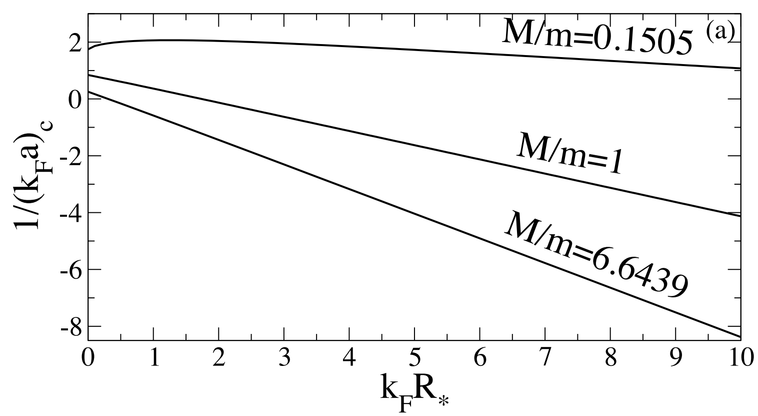
$$R_* = \frac{\pi \hbar^4}{\Lambda^2 \mu^2} \propto \frac{1}{\Delta B}$$

$\Delta B \equiv$ RESONANCE WIDTH

Conclusions

Simple variational ansatz limited to at most ONE pair of particle-hole excitations

Ground state: polaron-to-dimeron crossing point



Physical interpretation in terms of the Lamb shift

Conclusions

Non-trivial weakly interacting limit

$$\begin{cases} a \to 0^- \\ R_* \to +\infty \end{cases} \Rightarrow \boxed{s \equiv \frac{r}{1+r}(-aR_*)^{1/2}k_{\mathrm{F}}} \text{ FIXED}$$