Quasi-1D Bose gas revisited

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Outline

- 3D gas in harmonic potential is coherent below critical temperature
- 1D gas not always - quasicondensation phenomenon
- Classical Fields Approximation describes Bose gas at nonzero temperatures
- dependence of the quasicondensation on the interaction
- density matrix spectrum analysis for ideal gas in different geometries

BEC: Statistical picture — Inuitive picture

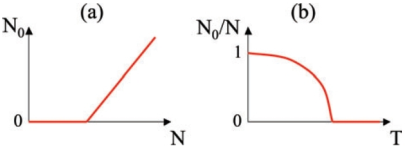
bosons:

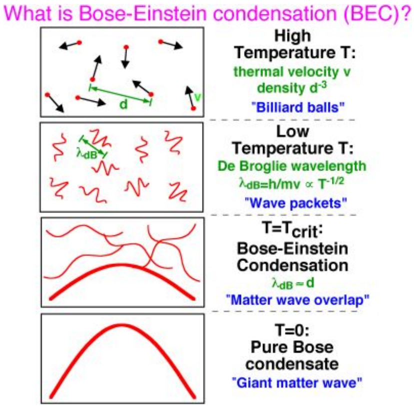
$$n(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

conservation of the number of particles:

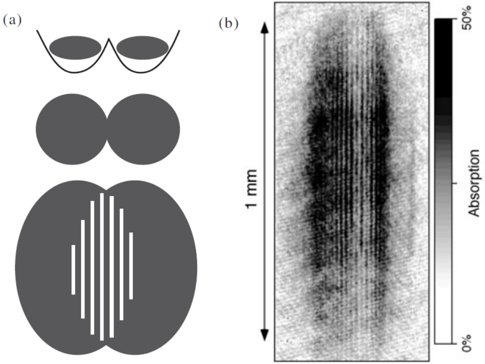
$$N_{ex} = \sum_{i=1}^{\infty} \frac{1}{e^{(\epsilon_i-\mu)/k_B T} - 1}$$

Phase transition at critical temperature T_c

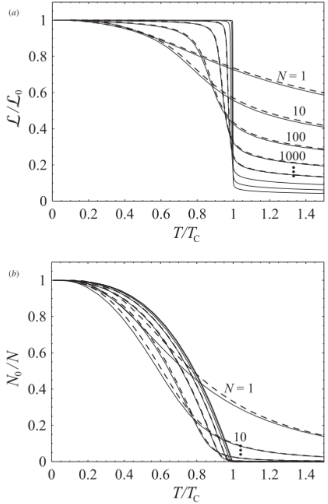




Ketterle, MIT



$$\mathcal{L}^2 = \frac{\int \mathrm{d}r_1 \int \mathrm{d}r_2 \, |\langle \hat{\psi}^\dagger(r_1)\hat{\psi}(r_2)\rangle|^2 (x_1 - x_2)^2}{\int \mathrm{d}r_1 \int \mathrm{d}r_2 \, |\langle \hat{\psi}^\dagger(r_1)\hat{\psi}(r_2)\rangle|^2}$$



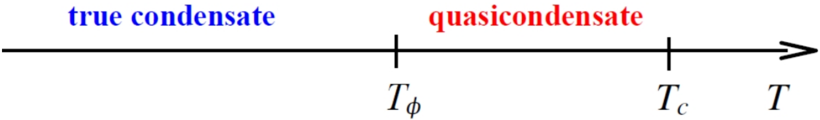
- interference with itself
- interference of two condensates

S. Barnett, Journal of Physics B, 33, 4177 (2000)

3D is coherent!

What about 1D?

Quasicondensation phenomenon



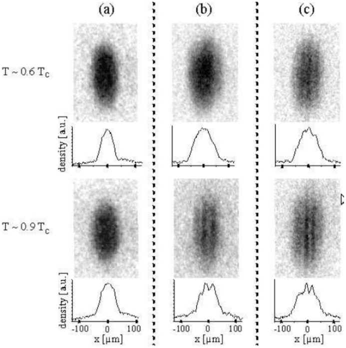
true condensate:

- coherence length $\approx$ size of the cloud

quasicondensate:

- coherence length $<$ size of the cloud
- large phase fluctuations
- small density fluctuations

Phase fluctuations in quasi-1D



[$\lambda = 10$ (a), 26 (b), 51 (c)]

D. S. Petrov et al., Phys. Rev. Lett. 85, 3745 (2000)
D. S. Petrov et al., Phys. Rev. Lett. 87, 050404 (2001)
S. Dettmer at al., Phys. Rev. Lett. 87, 160406 (2001)

Classical Field Approximation

$$\hat{H} = \int \hat{\Psi}^\dagger(x) \left[\frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 \right] \hat{\Psi}(x) dx + \frac{g}{2} \int \hat{\Psi}^\dagger(x)\hat{\Psi}^\dagger(x)\hat{\Psi}(x)\hat{\Psi}(x)$$

Base - harmonic eigenstates: $\hat{\Psi}(x) = \sum_n \phi_n(x)\hat{a}_n$

Long wavelength part of Bose atomic field replaced by a classical function:

$\hat{a}_n, \hat{a}_n^\dagger \mapsto \alpha_n, \alpha_n^*$ $\qquad \Psi(x) = \sum_{n=0}^{K} \phi_n(x)\alpha_n$

Close analogy to electromagnetic field.

The most relevant question:

Where to cut?

Method

- We have a finite dimensional classical system!

$$E(\{\alpha_n\}) = \hbar\omega \sum_{n=0}^{K} n|\alpha_n|^2 + E_{int}(\{\alpha_n\})$$

- Probability distribution in canonical ensemble:

$$P(\{\alpha_n\}) = \frac{1}{Z(N,T)} \exp\left[-\frac{E(\{\alpha_n\})}{k_B T}\right]; \qquad \sum_{n=0}^{K} |\alpha_n|^2 = N$$

- **Metropolis Monte Carlo algorithm** may be used to generate probability distribution!

- Identification of the condensate requires diagonalization of the single particle density matrix:

$$\rho(x,y) = \langle \Psi(x)^* \Psi(y) \rangle = \sum_{i,j} \langle \alpha_i^* \alpha_j \rangle \phi_i^*(x) \phi_j^*(y)$$

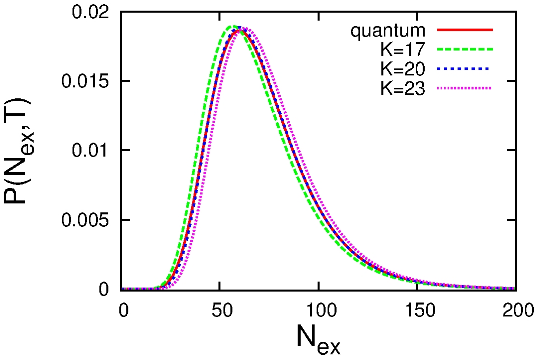
Quantum:

$$P(N_{ex}, T) = \frac{1}{\xi^{N_{ex}}} \prod_{l=N_{ex}+1}^{N} \left(1 - \xi^{l}\right)$$

Classical fields:

$$P_{cl}(N_{ex}, T) = \frac{1}{1-\xi^{N}} \left(\frac{1-\xi^{N_{ex}}}{1-\xi^{N}}\right)^{K-1}$$

$N = 200$, $T = 20\hbar\omega_0/k_B$



Optimal cut-off formula for ideal gas

$$\hbar\omega_0 K = k_B T$$

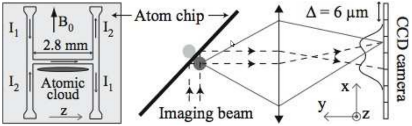
- repulsive gas

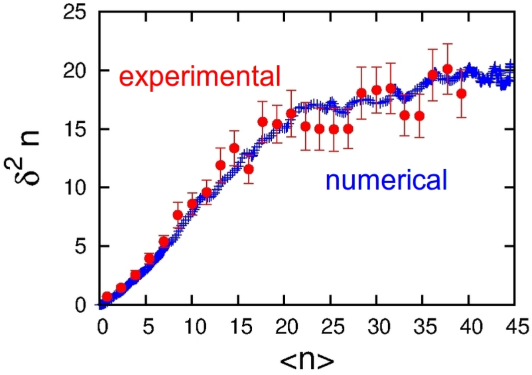
$$\hbar\omega_0 K = k_B T + \mu$$

- attractive gas

$$\hbar\omega(N, T) K = k_B T$$

P. Bienias et al., Physical Review A, 106, 135301 (2011)
P. Bienias et al., Europhysics Letter, 96, 10011 (2011)





$g_{1D} = 2\hbar\omega_\perp a$, $a = 5.7\text{nm}$,
$\omega_\perp = 2\pi \times 3.9$ kHz, $\omega_z = 2\pi \times 4\text{Hz}$,
$T = 0.09\hbar\omega_\perp/k_B = 88\hbar\omega_z/k_B$

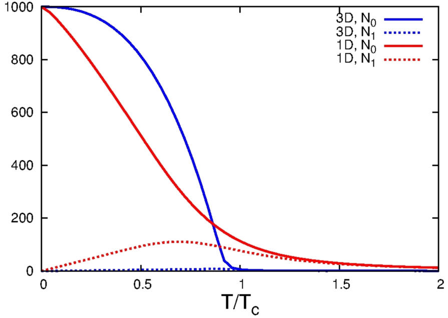
J. Armijo et al., Phys. Rev. A 83, 021605(R) (2011)

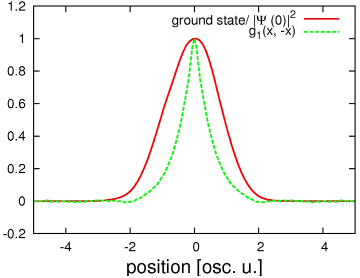
true condensate:

- $l_\phi \approx L$
- $N_0 \gg N_1$

quasicondensate:

- $l_\phi < L$
- large phase fluctuations
- small density fluctuations
- $N_0 \geq N_1 \geq N_2 \geq \ldots$



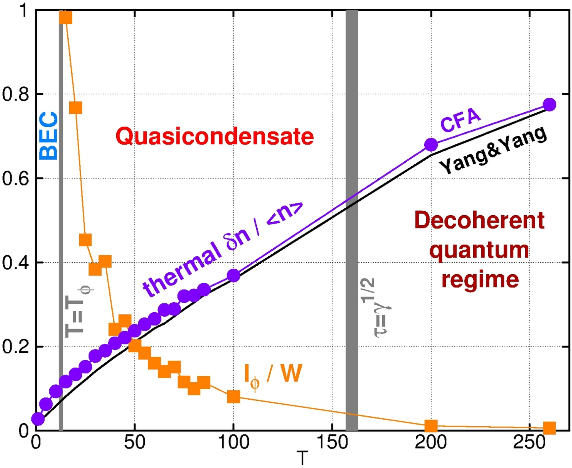


$$g^1(z,-z) = \frac{\langle \Psi(z)^* \Psi(-z) \rangle}{\langle |\Psi(z)|^2 \rangle}$$

1D: $T_c = \frac{\hbar\omega}{k_B} \frac{N}{\log(2N)}$, **3D**: $T_c = \frac{\hbar\omega}{k_B} \left(\frac{N}{\zeta(3)} \right)^{1/3}$

Density fluctuations are not so small

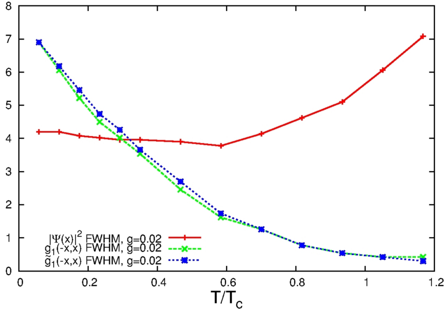
$$N{=}1000,\ \omega_Z = 2\pi \times 10Hz, g = 0.31\hbar\omega_Z(\hbar/m\omega_Z)^{1/2}$$



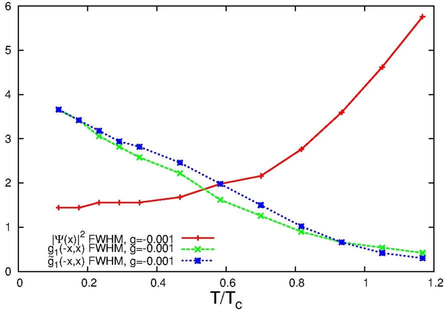
T. Karpiuk et al., arXiv:1205.2363 (2012)

Coherence length shortening regardless interaction

repulsive 1D gas



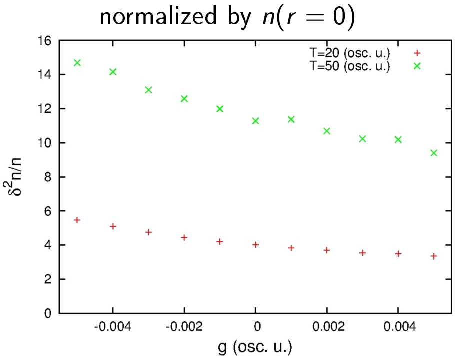
attractive 1D gas

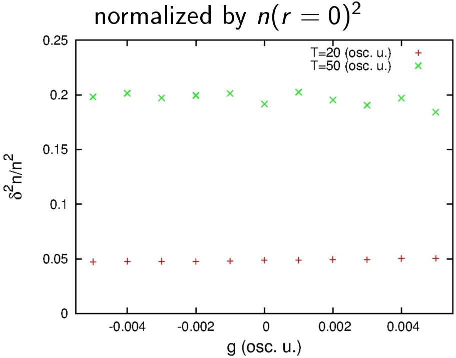


$$g_1(-x,x) = \frac{\langle \Psi^*(-x)\Psi(x)\rangle}{\langle |\Psi(x)|^2\rangle} = \frac{\langle \sqrt{n(-x)}e^{-i\phi(-x)}\sqrt{n(x)}e^{i\phi(x)}\rangle}{\langle n(x)\rangle}$$

$$\tilde{g}_1(-x,x) = \langle e^{-i\phi(-x)}e^{i\phi(x)}\rangle$$

P. Bienias et al., arXiv:1203.1811 (2012)





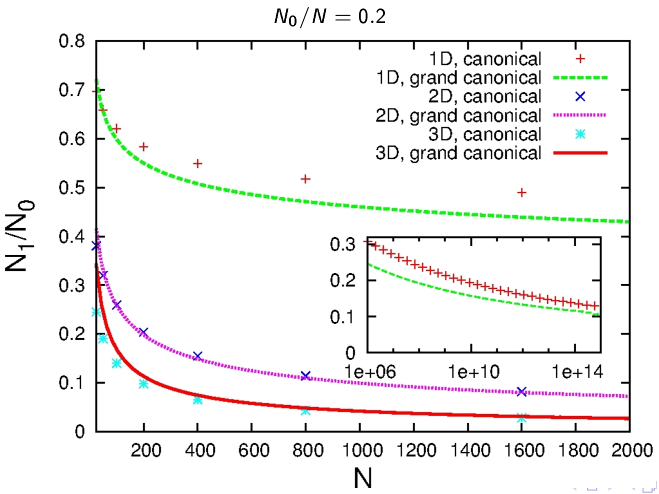
Let's concentrate on the ideal gas!

'Sticking' of N_1/N_0 for different dimensions

quasicondensate: $N_0 \geq N_1 \geq N_2 \geq \ldots$ **true condensate:** $N_0 \gg N_1$

canonical ensemble:

$Z_0 = 1;\quad Z_1(\beta) = \sum_\nu exp(-\beta\epsilon_\nu);\quad Z_N(\beta) = \frac{1}{N}\sum_{n=1}^{N} Z_1(n\beta)Z_{N-n}(\beta)$



Grand canonical ensemble:

1D:

$$\lim_{N\to\infty} \frac{N_1}{N_0} \sim \frac{1}{\ln(N)},$$

2D:

$$\lim_{N\to\infty} \frac{N_1}{N_0} \sim \frac{1}{N^{1/2}},$$

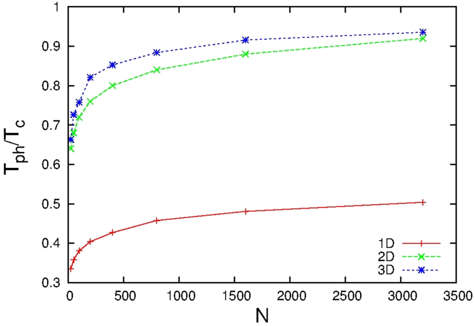
3D:

$$\lim_{N\to\infty} \frac{N_1}{N_0} \sim \frac{1}{N^{2/3}}$$

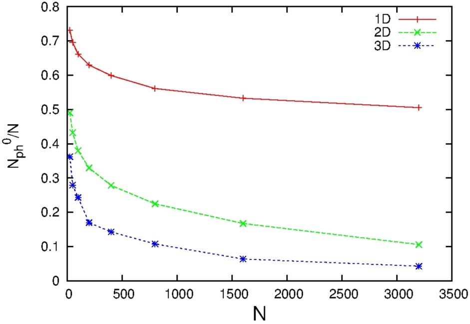
Quasicondensation transition

1D: $T_c = N/\log(2N)$, **2D**: $T_c = (\frac{N}{\zeta(2)})^{1/2}$, **3D**: $T_c = (\frac{N}{\zeta(3)})^{1/3}$

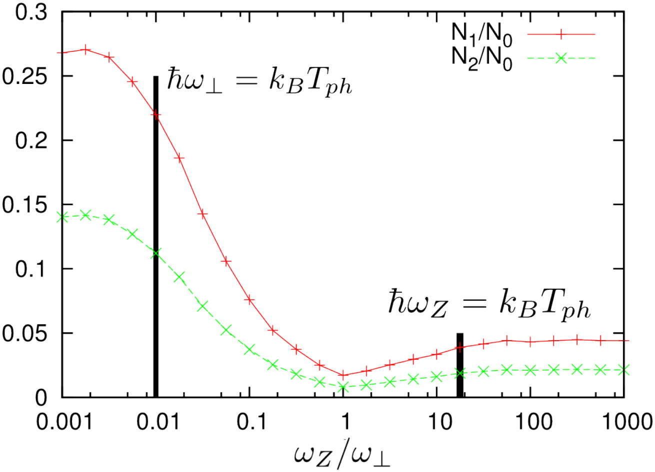
Temperature at the transition point



N_0/N at the transition point



$N = 1000,\ N_0 = 0.4$



P. Bienias et al., arXiv:1203.1811 (2012)

Summary

- 3D gas is fully coherent up to T_c
- Classical fields describe interacting gas very well
- 1D differs from 3D - quasicondensation phenomenon
- quasicondensation is a dimensional effect

P. Bienias, K. Pawłowski, M. Gajda, K. Rzążewski: *Quasicondensation reexamined*, arXiv:1203.1811v2 (2012)

P. Bienias, K. Pawłowski, M. Gajda, K. Rzążewski: *Statistical properties of one dimensional Bose gas*, Physical Review A, 106, 135301 (2011)

P. Bienias, K. Pawłowski, M. Gajda, K. Rzążewski: *Statistical properties of one dimensional attractive Bose gas*, Europhysics Letter, 96, 10011 (2011)