# Spinor dynamics in a multi-component Fermi gas

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# Spinor dynamics in a multi-component Fermi gas

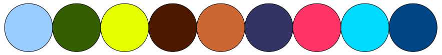
## Outline

- Description by density matrix / Wigner function
- Collisionless regime (mean field)
- Spinor dynamics
- Collisional approach (extension to mean field)
- More spinor dynamics
- Conclusions and outlook

# Spinor gases

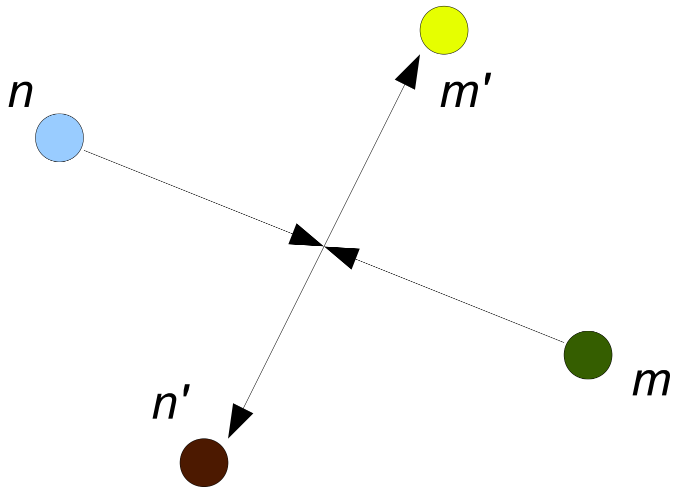
<u>Overview and motivation</u>

Spinor gas: Spin *F*, **2*F*+1** internal states
..., $m = -3/2$, $m = -1/2$, $m = 1/2$, $m = 3/2$, …



...

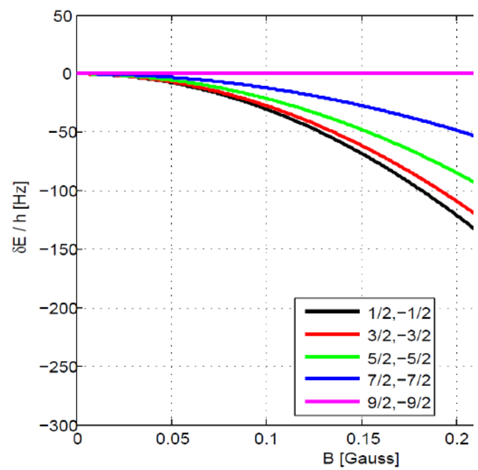
Collisions preserve **total** spin → more than 2 components lead to spinor dynamics




$$m + n = m' + n'$$

Internal states after the collision can be different than before.

**Spinor dynamics = population transfer**




Quadratic Zeeman effect

$$E_{\mathrm{QZE}} = qm^2$$

→ Zeeman energy not conserved

# Trapped spinor fermi system

Hamiltonian and density matrix

$$\hat{H} = \hat{H}_0 + \hat{H}_1$$

Single particle: $$\hat{H}_0 = \int \mathrm{d}^3 r \sum_{mn} \hat{\psi}^\dagger_m(\vec{r}) \left[ -\frac{\hbar^2}{2M}\nabla^2 + V^{\mathrm{ext}}_{mn}(\vec{r}) \right] \hat{\psi}_n(\vec{r})$$

Two particle: $$\hat{H}_1 = \frac{1}{2}\sum_{ijkl}\int \mathrm{d}^3 r U_{ijkl} \hat{\Psi}^\dagger_i(\vec{r}) \hat{\Psi}^\dagger_k(\vec{r}) \hat{\Psi}_l(\vec{r}) \hat{\Psi}_j(\vec{r})$$

$$V^{\mathrm{ext}}_{mn} = \frac{1}{2} M\omega^2 \vec{r}^2 \delta_{mn} + pS^z_{mn} + q(S_z)^2_{mn}$$

S-wave-scattering, weak interactions:

$$U_{ijkl} = \sum_{S=0,2,\ldots}^{2F-1} \sum_{M=-S}^{S} g_S \langle ik|SM\rangle\langle SM|jl\rangle \qquad g_S = 4\pi\hbar^2 a_S/M$$

We describe the system and its time evolution with the **single-particle-density-matrix**

$$\rho_{mn}(x,y) = \left\langle \hat{\psi}^\dagger_m(x)\hat{\psi}_n(y) \right\rangle \qquad i\dot{\rho}_{mn}(x,y) = \left\langle \left[ \hat{\psi}^\dagger_m(x)\hat{\psi}_n(y), \hat{H} \right] \right\rangle$$

# Wigner function

Definition: $$W_{mn}(\vec{x},\vec{p}) = \int \frac{\mathrm{d}^3 y}{(2\pi\hbar)^3} \mathrm{e}^{-i\vec{p}\cdot\vec{y}} \rho_{mn}(\vec{x}-\vec{y}/2, \vec{x}+\vec{y}/2)$$

Advantages: Knowing *W* we can extract many observables by integration / tracing
Suited for collisional methods

1) In phase-space: Thomas-Fermi distribution, exact for non-interacting gas.

$$W_0(\vec{r},\vec{p}) = \frac{1}{(2\pi\hbar)^3}\left\{\exp\left[\frac{1}{k_\mathrm{B}T}\left(\frac{\vec{p}^2}{2M}+\frac{1}{2}M\omega^2\vec{r}^2-\mu\right)\right]+1\right\}^{-1}$$

2) In spin space: Lots of freedom to create spin states. Examples:

Mixed state (incoherent): $|+\frac{1}{2}\rangle \quad |-\frac{1}{2}\rangle$

Pure state (coherent): $|+\frac{1}{2}\rangle + e^{i\phi}|-\frac{1}{2}\rangle$

$$\hat{W}_{\mathrm{mixed}} = \frac{1}{2}\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & 0 & 0 & \cdots \\ \cdots & 0 & 0 & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\hat{W}_{\mathrm{pure}} = \frac{1}{2}\begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \\ \cdots & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & 1 & e^{i\phi} & 0 & \cdots \\ \cdots & 0 & e^{-i\phi} & 1 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & 0 & \cdots \\ & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

# Equation of motion

Von Neumann-equation:

$$i\dot{\rho}_{mn}(x,y) = \left\langle \left[ \hat{\psi}^\dagger_m(x)\hat{\psi}_n(y), \hat{H} \right] \right\rangle$$

Wick decomposition (mean field or Hartree-Fock approximation)

$$\left\langle \hat{\psi}^\dagger_k \hat{\psi}^\dagger_l \hat{\psi}_m \hat{\psi}_n \right\rangle \approx \langle \hat{\psi}^\dagger_k \hat{\psi}_n \rangle \langle \hat{\psi}^\dagger_l \hat{\psi}_m \rangle - \langle \hat{\psi}^\dagger_k \hat{\psi}_m \rangle \langle \hat{\psi}^\dagger_l \hat{\psi}_n \rangle$$

Mean field → effective Potential

Exchange interaction

$$V^{\mathrm{mf}}_{mn}(\vec{r}) = \int \mathrm{d}^3 q \sum_{kl} \left( U_{mnkl} - U_{mlkn} \right) W_{kl}(\vec{r},\vec{q})$$

$$V_{mn}(\vec{r}) = V^{\mathrm{ext}}_{mn}(\vec{r}) + V^{\mathrm{mf}}_{mn}(\vec{r})$$

Quantum Liouville equation:

$$\dot{W}_{mn}(\vec{r},\vec{p}) = -\frac{\vec{p}}{M}\cdot\nabla_r W_{mn}(\vec{r},\vec{p}) + \frac{1}{i\hbar}\sum_{\alpha=0}^{\infty}\frac{1}{\alpha!}\left[\frac{i}{2}\nabla_y\cdot\nabla_p\right]^{\alpha}\left(V_{mk}(\vec{y})W_{nk}(\vec{r},\vec{p}) - (-1)^{\alpha}V_{kn}(\vec{y})W_{km}(\vec{r},\vec{p})\right)|_{\vec{y}=\vec{r}}$$

Semiclassical approximation for coordinates (not spin!):

$$\dot{\mathrm{W}}(\vec{r},\vec{p}) = -\frac{\vec{p}}{M}\cdot\nabla_r \mathrm{W}(\vec{r},\vec{p}) + \frac{1}{i\hbar}\left[\mathrm{W}(\vec{r},\vec{p}), \mathrm{V}^T(\vec{r})\right] + \frac{1}{2}\nabla_p\cdot\nabla_y \left\{\mathrm{W}(\vec{r},\vec{p}), \mathrm{V}^T(\vec{y})\right\}|_{\vec{y}=\vec{r}}$$

Spin-mean-field (leading order) mean-field correction to trap

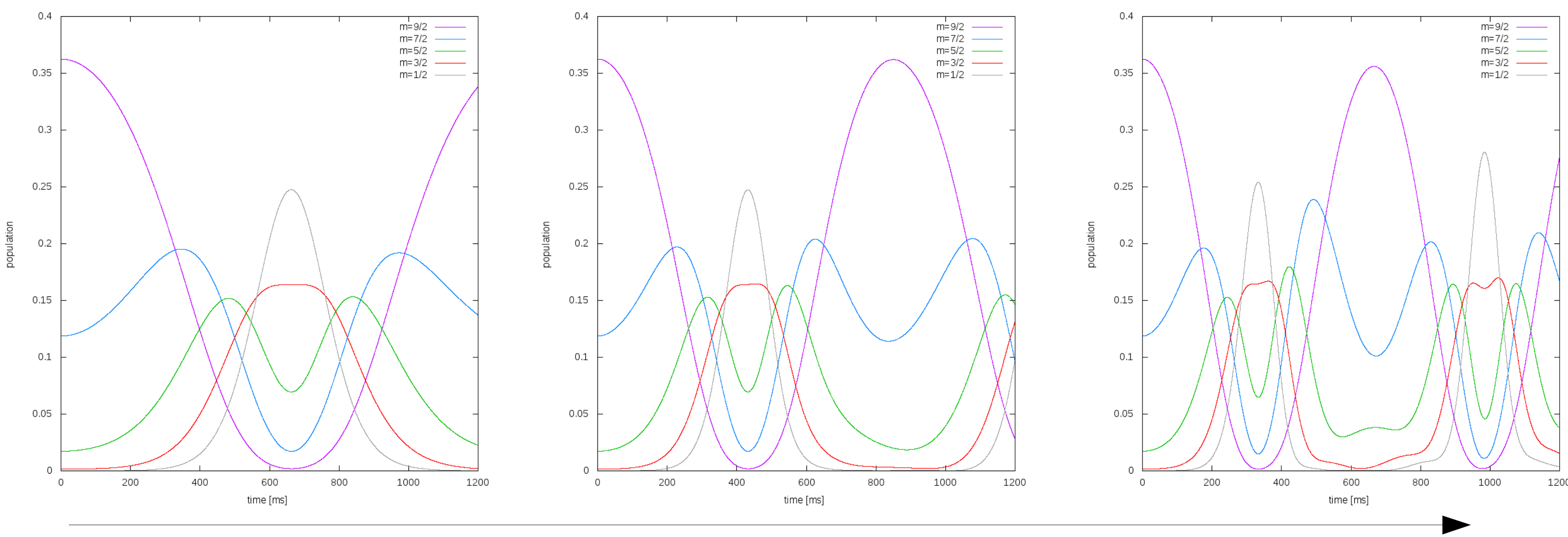
# Coherent spinor dynamcis

**Coherent** population transfer, described by mean-field theory.
Has been also observed in spinor BEC.

Parameters: Initial coherences, scattering lengths, number of states, QZE,...
Many possibilities.

Here: $F$=9/2, initial state coherent superposition of $m$=±9/2, ±7/2, ±5/2

Oscillatory modes




Magnetic field

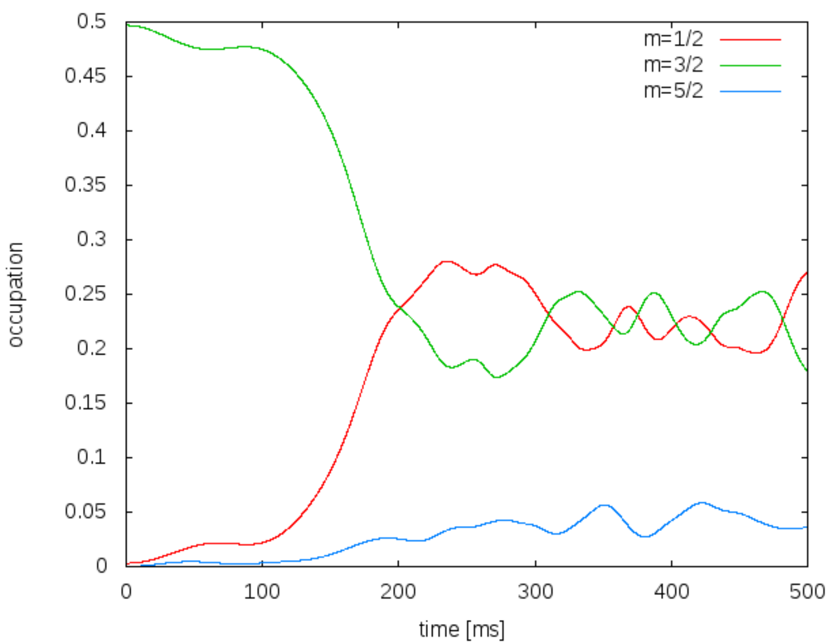
Frequency ~ QZE

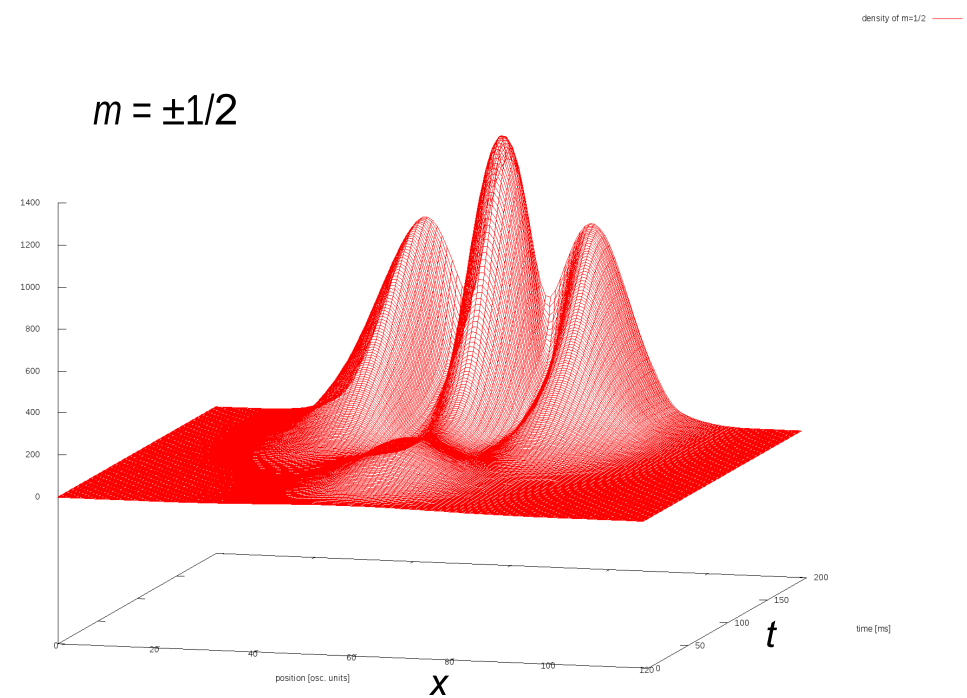
# Coherent spinor dynamcis

**Coherent** population transfer, described by mean-field theory.

Here: *F* = 5/2, initial state *m* = ±3/2, small seed in *m* = ±1/2

Exponential modes



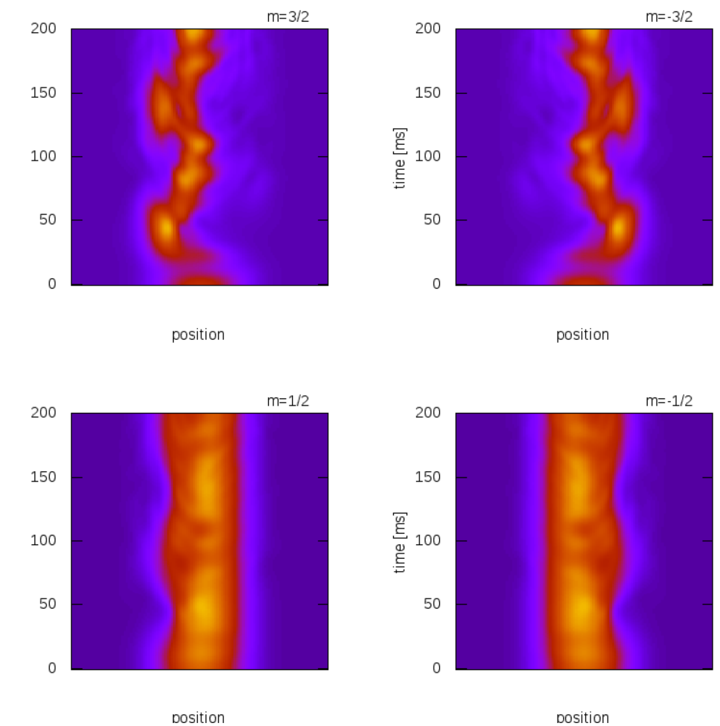

Feature:
*m*=±5/2 does not participate, can create lower spin subsystem

Formation of spatial structures:
Interplay of orbital and spin degrees of freedom

# Spin waves

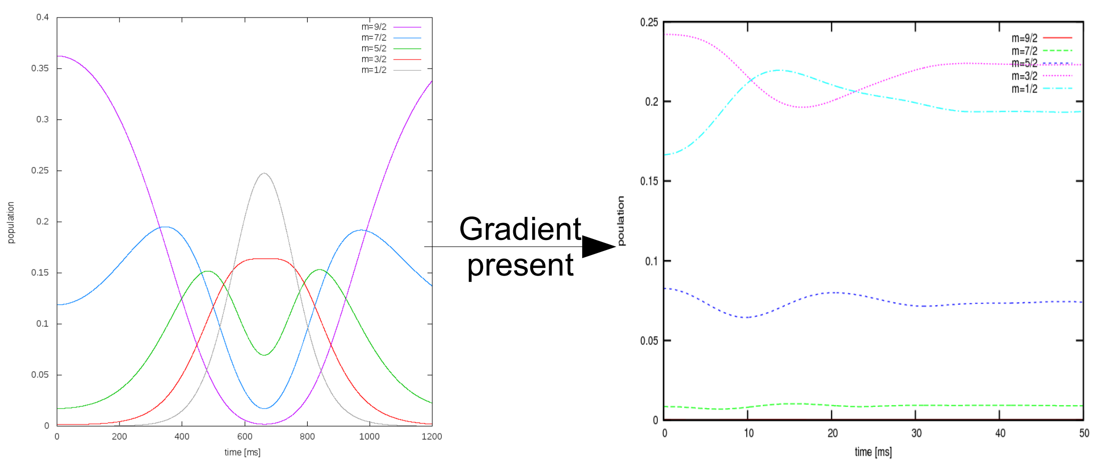
Collective excitations arising from exchange interaction.
Spatial movement of spin components. Described by mean-field approach.

Dipole oscillations for F=3/2




**Coherent** states are very susceptible to magnetic field **gradient**
Gradient displaces spin components in the trap

Problem: Spatial separation **reduces** spinor dynamics
Spin waves easy to excite, hard to get rid of




Outlook: Interesting to study for higher spins due to presence of higher magnetic multipoles.

# More spinor dynamics

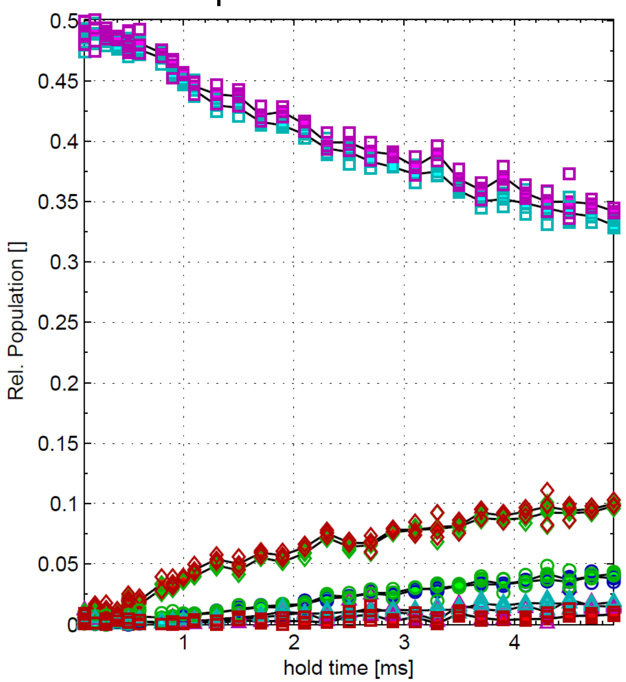
Is a mean-field approach good enough?

For a mixed initial state, mean-field predicts no spinor dynamics.
Coherence (off-diagonal elements) needed.

$$\dot{\mathrm{W}}(\vec{r},\vec{p}) = -\frac{\vec{p}}{M}\cdot\nabla_r \mathrm{W}(\vec{r},\vec{p}) + \frac{1}{i\hbar}\left[\mathrm{W}(\vec{r},\vec{p}), \mathrm{V}^T(\vec{r})\right] + \frac{1}{2}\nabla_p\cdot\nabla_y \left\{\mathrm{W}(\vec{r},\vec{p}), \mathrm{V}^T(\vec{y})\right\}|_{\vec{y}=\vec{r}}$$

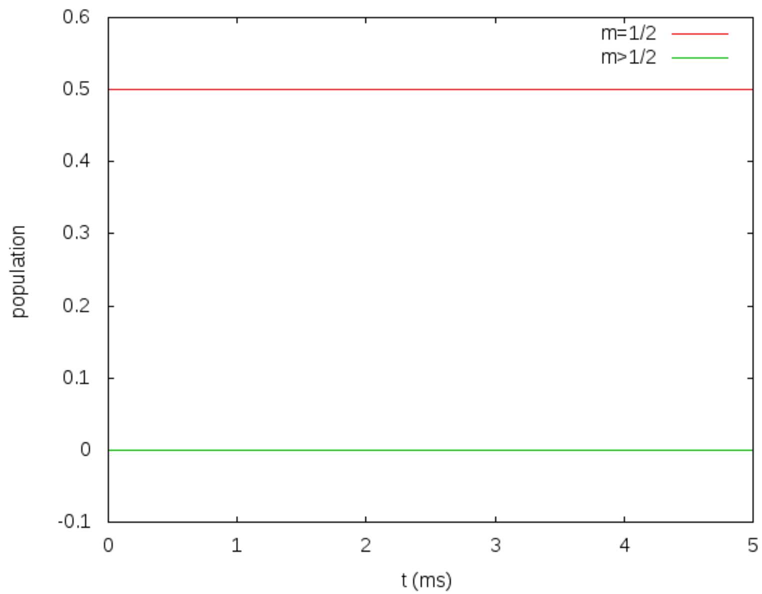
Vanishes for incoherent states

Experimental data, trapped 40K, *F*= 9/2
incoherent spin mixture *m* = ±1/2




Courtesy of Sengstock group

Mean field theory predicts:

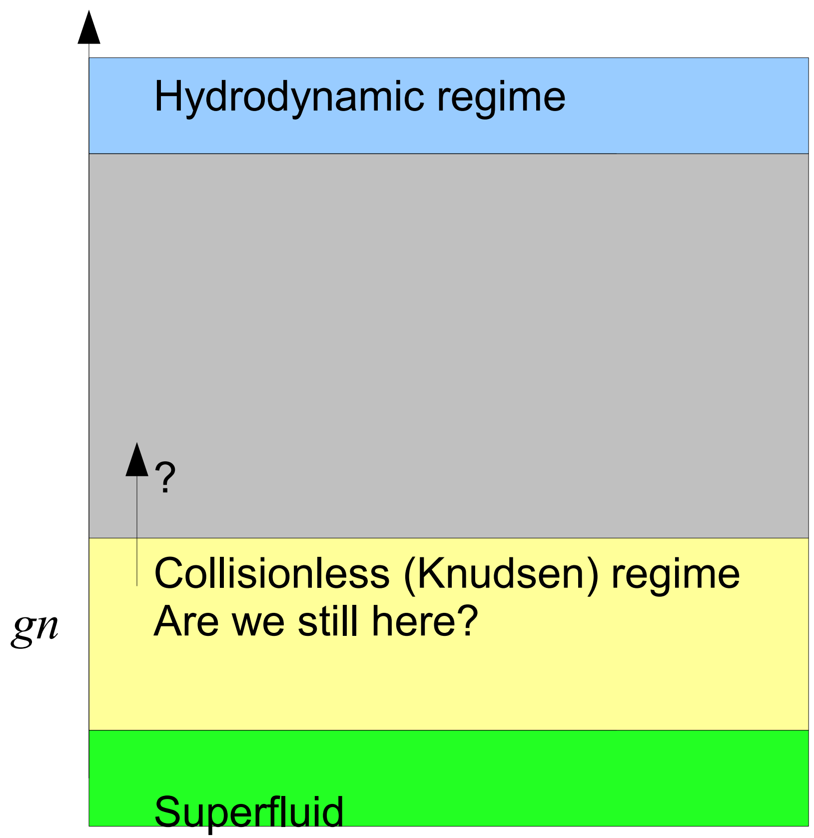


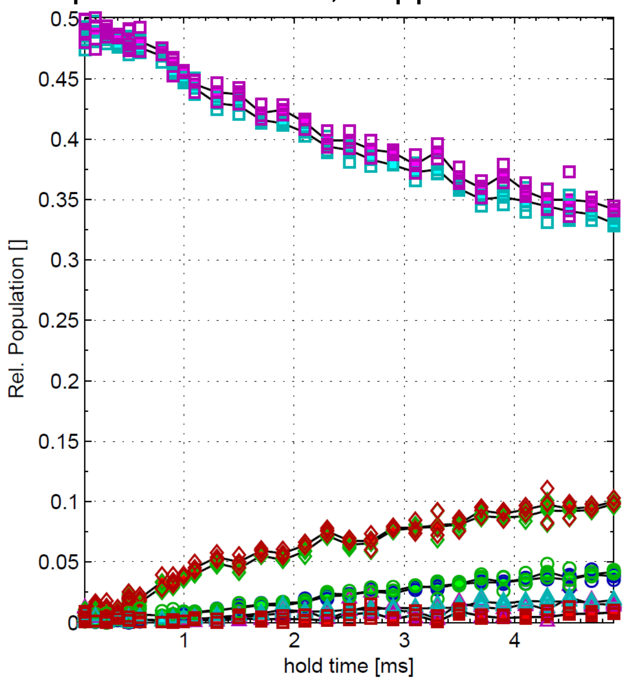
# Collisional approach

Is a mean-field approach good enough?

Equation is a collisionless Boltzmann equation

$$\dot{\mathrm{W}}(\vec{r},\vec{p}) = -\frac{\vec{p}}{M}\cdot\nabla_r \mathrm{W}(\vec{r},\vec{p}) + \frac{1}{i\hbar}\left[\mathrm{W}(\vec{r},\vec{p}),\mathrm{V}^T(\vec{r})\right] + \frac{1}{2}\nabla_p\cdot\nabla_y\left\{\mathrm{W}(\vec{r},\vec{p}),\mathrm{V}^T(\vec{y})\right\}|_{\vec{y}=\vec{r}}$$




Looks like relaxation to equilibrium

# Collisional approach

Correction to mean-field approach

Boltzmann equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} + \frac{1}{i\hbar}\left[\hat{\rho}, \hat{H}_0\right] = \hat{I}^{\mathrm{coll}}$$

J. N. Fuchs, D. M. Gangardt and F. Laloë, Eur. Phys. J. D 25, 57 (2003)

R.h.s: “Collisional Integral”, change of density-matrix due to collisions
Many approaches possible, we choose the Lhuillier-Laloë – Ansatz (not the only one!)

$$\hat{I}^{\mathrm{coll}} \approx \frac{\hat{\rho}\prime - \hat{\rho}}{\Delta t}$$

Change of the **single-particle** density matrix
*Δt* small, but still longer than duration of collisions

A collision is a **two-particle** process, we know what happens to the **two-particle** density matrix

$$\hat{\rho}^{(2)}(1,2) \rightarrow \hat{S}\hat{\rho}^{(2)}(1,2)\hat{S}^{\dagger}$$

(Heisenberg S-matrix)

L.-L.: No entanglement before **and** after the collision - Boltzmann's molecular chaos (Stosszahlansatz)

$$\hat{\rho}^{(2)}(1,2) \approx \hat{\rho}(1) \otimes \hat{\rho}(2)$$

$$\hat{S}\hat{\rho}^{(2)}(1,2)\hat{S}^{\dagger} \approx \hat{S}\hat{\rho}(1) \otimes \hat{\rho}(2)\hat{S}^{\dagger}$$

Why? Many-particle system.
No repeated collisions between same particles

Two-particle situation, *F*=9/2:
Krauser et al. ArXiv 1203.0948 (2012)

# Collisional approach

Collision integral

S-matrix to T-matrix: $\hat{S} = \hat{1} - 2\pi i \hat{T}$

Wigner transform everything, get terms linear and quadratic in T: $\hat{I}^{\text{coll}} = \hat{I}^{\hat{T}} + \hat{I}^{\hat{T}^2}$

Expand T-matrix in powers of the scattering lengths:

$$T_S = (2\pi)^{-3}\left(g_S - \frac{iM|\vec{k}|}{4\pi\hbar^2} g_S^2\right) + \dots$$

First order reproduces the mean-field equation of motion

$$I_{ij}^{\text{MF}}(\vec{r},\vec{p}) = \frac{1}{i\hbar}\int \mathrm{d}^3 q \sum_{abc} \left(U_{iacb} - U_{caib}\right) W_{bc}(\vec{r},\vec{q}) W_{aj}(\vec{r},\vec{p}) - \left(U_{ajbc} - U_{acbj}\right) W_{cb}(\vec{r},\vec{q}) W_{ia}(\vec{r},\vec{p})$$

Second order, beyond mean-field, includes momentum transfer

$$I_{ij}^{T}(\vec{r},\vec{p}) = \frac{1}{\hbar}\int \mathrm{d}^3 q \sum_{abc} \frac{|\vec{q}|}{2\hbar^2}\left(\tilde{U}_{iacb} W_{bc}(\vec{r},\vec{p}-\vec{q}) W_{aj}(\vec{r},\vec{p}) + \tilde{U}_{ajbc} W_{cb}(\vec{r},\vec{p}-\vec{q}) W_{ia}(\vec{r},\vec{p})\right)$$

$$I_{ij}^{T^2}(\vec{r},\vec{p}) = \int \mathrm{d}^3 q \sum_{abc} \frac{M|\vec{q}|}{4\pi\hbar^2} T_{ijabcd} W_{bc}(\vec{r},\vec{p} - \tfrac{1}{2}(|\vec{q}| + \Delta_{ikab})\vec{e}_q) W_{aj}(\vec{r},\vec{p} - \tfrac{1}{2}(|\vec{q}| - \Delta_{jkcd})\vec{e}_q)$$

Quadratic Zeeman-shift $\Delta_{ijkl} = q(i^2 + j^2 - k^2 - l^2)$

# Collisional dynamics

Relaxation induced by collisions. Long time scales

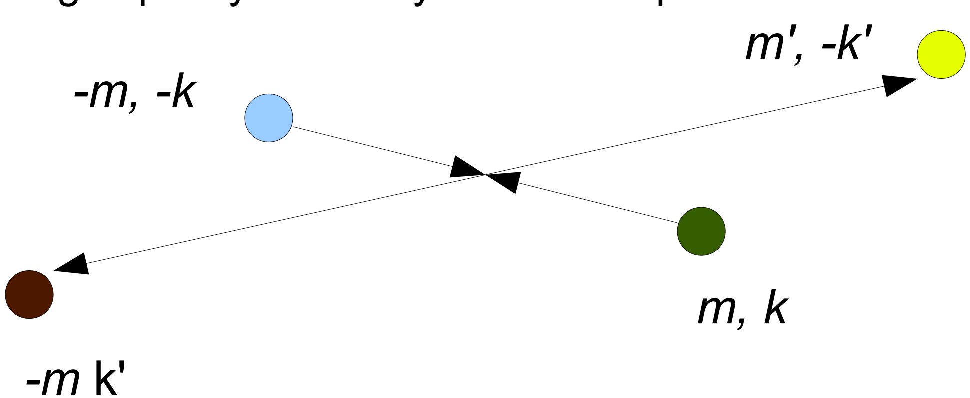
Incoherent process. Damps spin waves, coherent dynamics

Particles exchange momentum, restore system to equilibrium

Standard approach: Relaxation time approximation

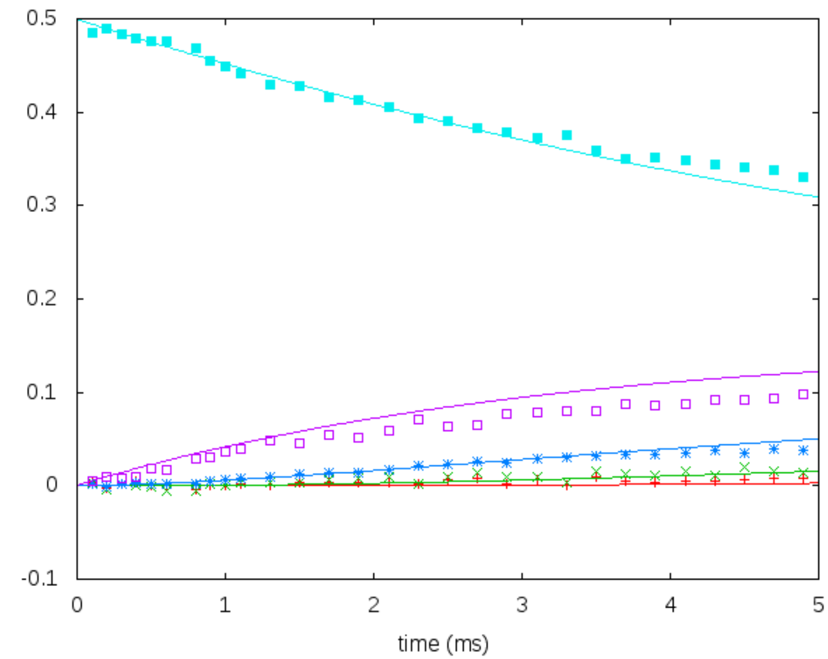
$$\hat{I}^{\mathrm{coll}} \approx \frac{\hat{W} - \hat{W}^{\mathrm{eq}}}{\tau}$$

High spin system may be too complicated

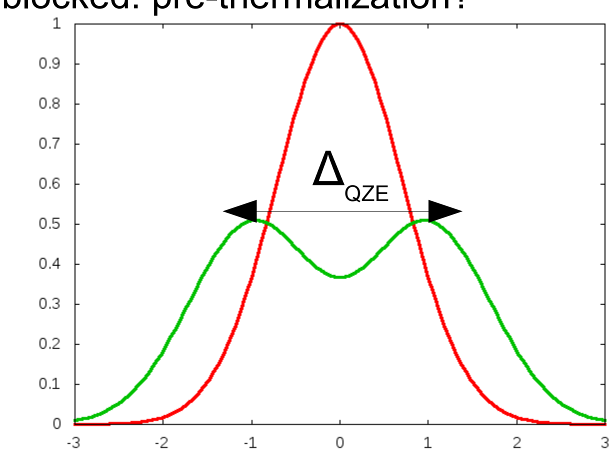



Collision in presence of QZE: $m^2 > m'^2$, $k' > k$

Comparison with experimental data




Momentum distribution, relaxation to equilibrium blocked: pre-thermalization?



# Conclusions

We have derived a **multi-component Boltzmann-equation** that combines

- **Mean field effects**
  - Coherent spinor dynamics
  - Spin waves
- **Collision effects**
  - Relaxation
  - Damping of coherent phenomena
  - Thermalization

in a trapped multi-component Fermi gas for a wide range of parameters

- Spin *F*, magnetic field, temperature, initial coherences, scattering lengths,...

and with good agreement with experiments.