Quantum Coherence via Smooth Optimal Control

Björn Bartels[1], Florian Mintert[1],
Tobias Nöbauer[2], Johannes Majer[2]

[1] Freiburg Institute for Advanced Studies
[2] Atominstitut Vienna

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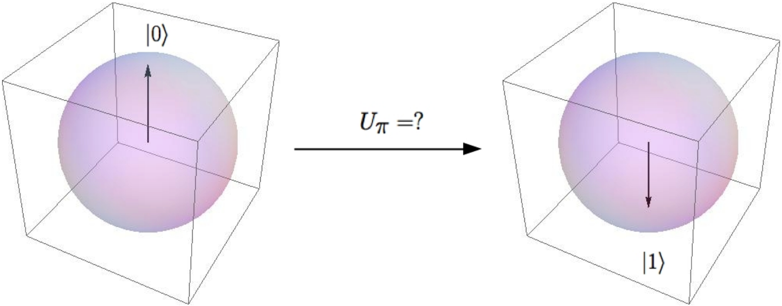
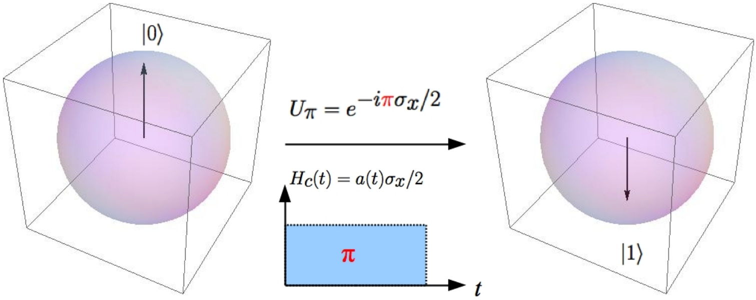
Smooth optimal control:

1. Why do we need it?
2. How does it work?
3. An example: control of spin ensembles

How to rotate a qubit?

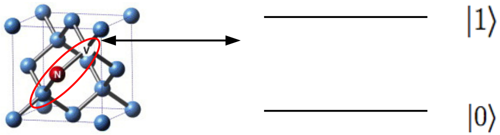


Apply a control pulse!



Why is it not that simple?

Nitrogen-Vacancy Centers in Diamond

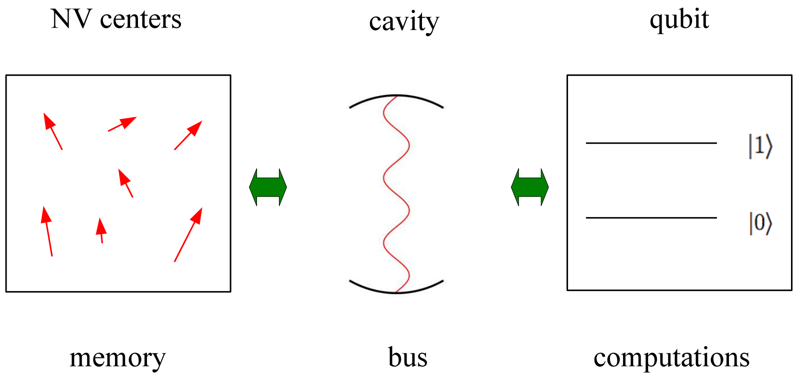


Pham *et al.*, New J. Phys. **13**, 045021 (2011)

Davies, Hamer, Proc. R. Soc. London A **348**, 285 (1967)

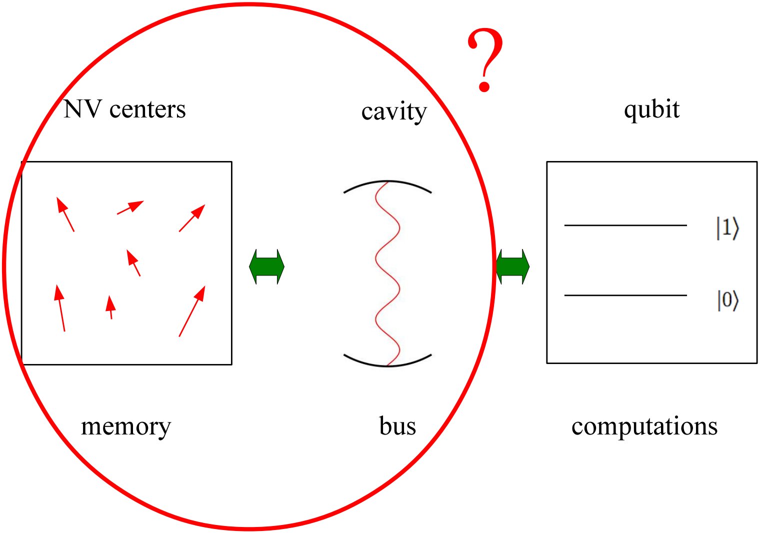
very long coherence times (up to ms): quantum memory

Quantum Computers



Kubo *et al.*, PRL **107**, 220501 (2011)

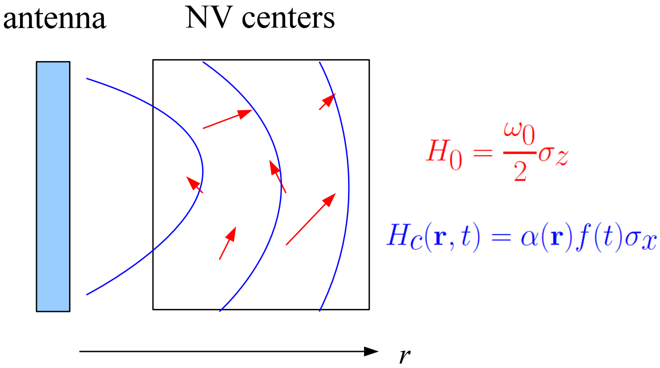
Quantum Computers



The Problem of Spin Ensembles

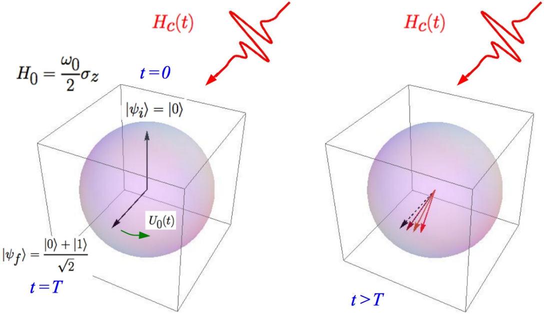
experiments in Vienna (Majer) and Paris (Estève):

Amsüss et al., PRL **107**, 060502 (2011) + Kubo et al., PRL **107**, 220501 (2011)



problems: inhomogeneous broadening of NV ensemble (→NMR), inhomogeneous field of antenna (new!)

Control of Spin Ensembles



first pulse:
map spins from $|\psi_i\rangle$ to $|\psi_f\rangle$

second pulse:
refocus dephased spins

optimal control copes with

- different spin frequencies ω_0
- different amplitudes α of the control field

Optimal Control

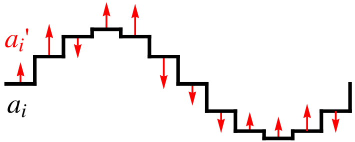
- objective of optimal control: maximize fidelity

$$\max \underbrace{\left\langle \left| \langle \psi_f | U_{f(t)}(T) | \psi_i \rangle \right|^2 \right\rangle_{\omega_0,\,\alpha}}_{\mathcal{F}[f(t)]}$$

- $$\frac{\delta \mathcal{F}[f(t)]}{\delta f(t)} = 0\,?$$

Pulse Shaping with GRAPE/Krotov

- piecewise constant functions:



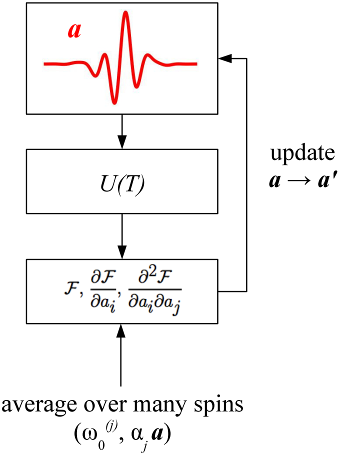
- iterative improvement:

$$\mathcal{F}_{\mathbf{a}+\delta\mathbf{a}} \approx \mathcal{F}_{\mathbf{a}} + \delta\mathbf{a} \cdot \nabla_{\mathbf{a}}\mathcal{F}_{\mathbf{a}} + \frac{1}{2}\sum_{i,j} \delta a_i \delta a_j \partial_{a_i} \partial_{a_j} \mathcal{F}_{\mathbf{a}}$$

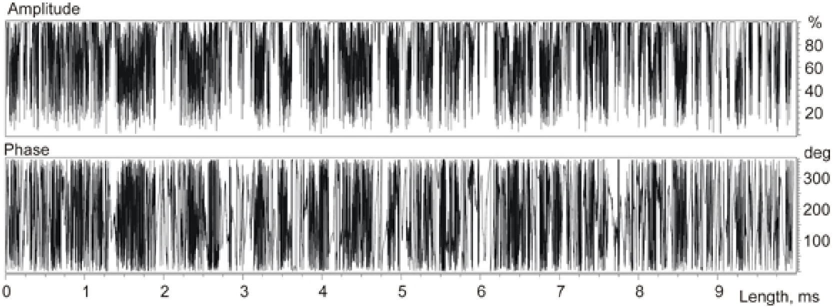
Glaser *et al.*, J. Magn. Reson. **172**, 296 (2005)

Krotov, *Global Methods in Optimal Control Theory*, Dekker (1995)

Optimization Algorithm



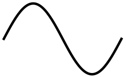
Typical GRAPE Pulse: High Frequency Components



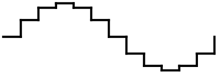
Kobzar et al., J. Magn. Reson. **173**, 229 (2005)

Our Approach: Smooth Control

alternative solution:



instead of



Caneva, Calarco, Montangero, PRA **84**, 022326 (2011)

Romero Isart, García Ripoll, PRA **76**, 052304 (2007)

$\Rightarrow$ use only a few frequency components:

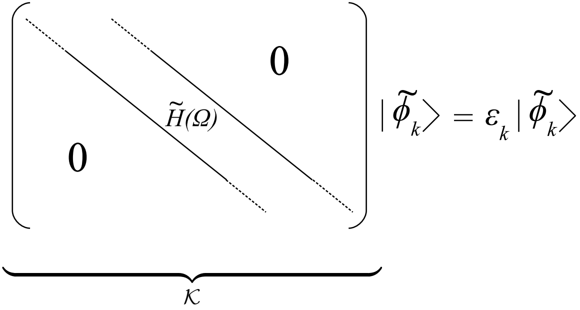
$$H_c(t) = f(t)\mathbf{h} \qquad f(t) = \sum_{k=1}^{n} a_k \sin(k\Omega t)$$

Techniques I: What is U(t)?

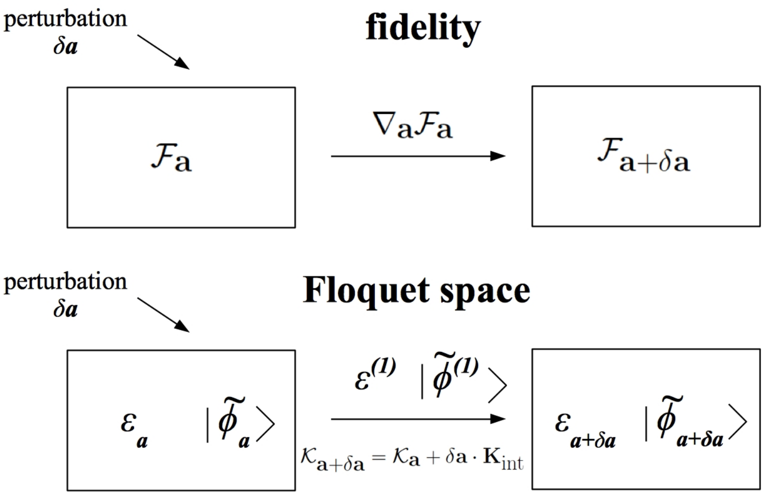
dynamics periodic in time:

$$i\partial_t|\psi_k(t)\rangle = H(t)|\psi_k(t)\rangle \qquad H(t+\tfrac{2\pi}{\Omega}) = H(t)$$

Floquet: $|\psi_k(t)\rangle = e^{i\epsilon_k t}|\phi_k(t)\rangle$ with $|\phi_k(t+\frac{2\pi}{\Omega})\rangle = |\phi_k(t)\rangle$
$\Rightarrow$ eigenvalue problem in Fourier space:



Techniques II: What is $\nabla_{\mathrm{a}}\mathcal{F}_{\mathrm{a}}$?



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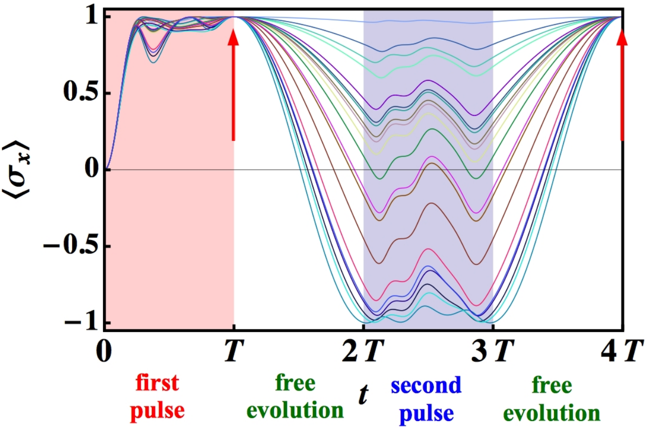
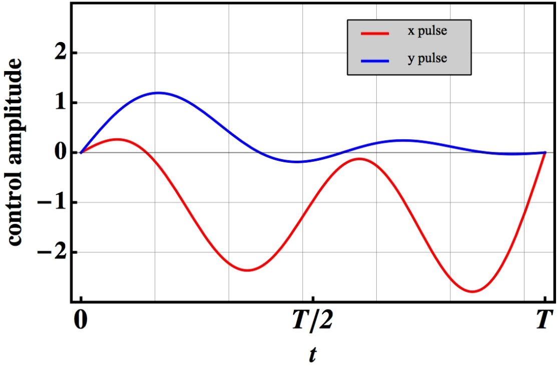
Robustness against Different Spin Frequencies

broad frequency interval: here $\Delta\omega \cdot T = \pi$

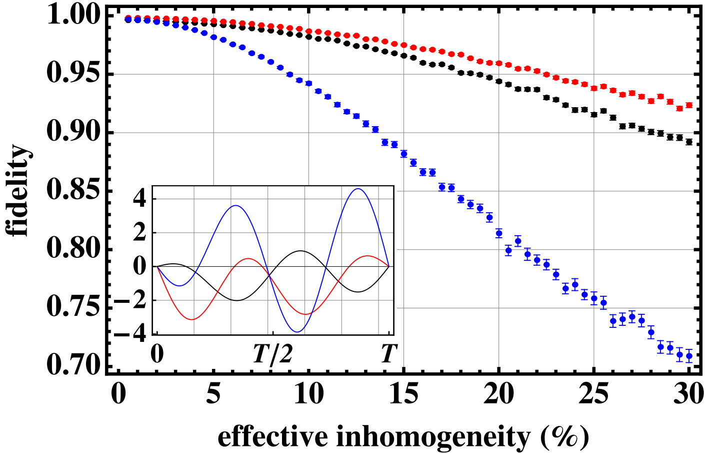


Typical Control Pulse: Only 4 Frequency Components



achieved fidelities: > 99.99 % for the previous example

Robustness against Inhomogeneity in the Control Field



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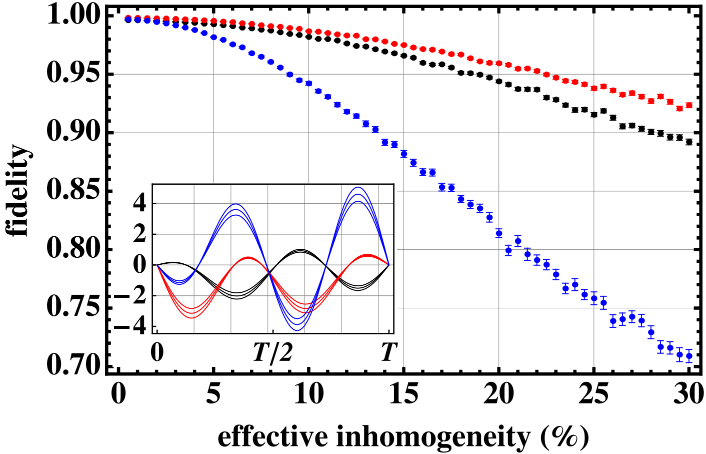
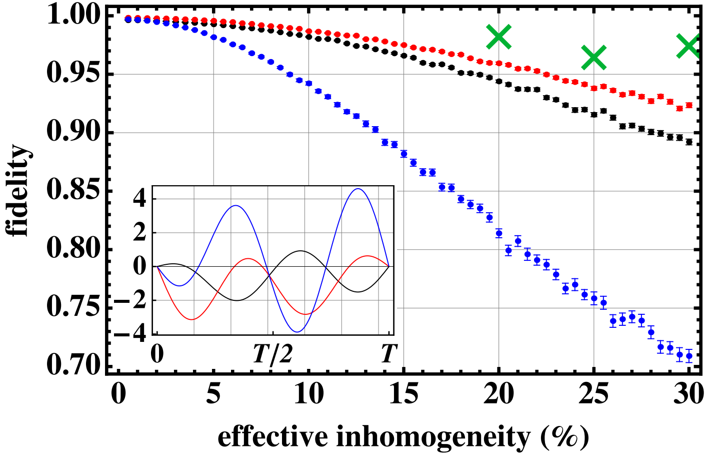
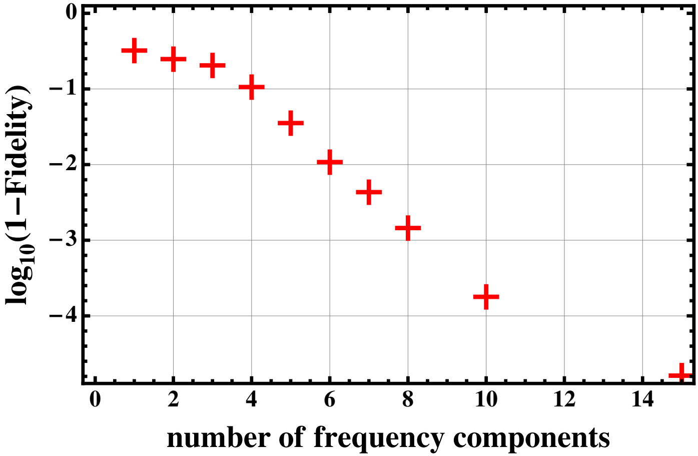


Robustness against Inhomogeneity in the Control Field



The more frequency components, the higher the fidelity

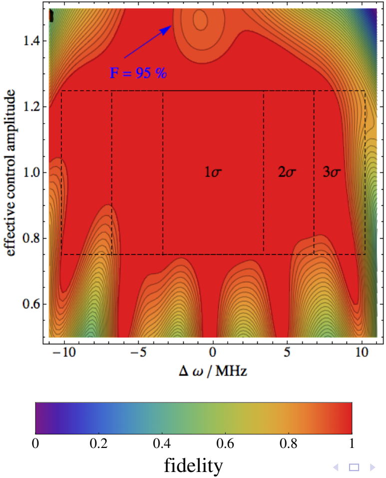
$\Delta\omega = \frac{1}{200\,\text{ns}}$, $T = 3.14\,\mu$s, $\Omega = 1\,$MHz, 20 % of inhomogeneity (only $\pi/2$-pulse)



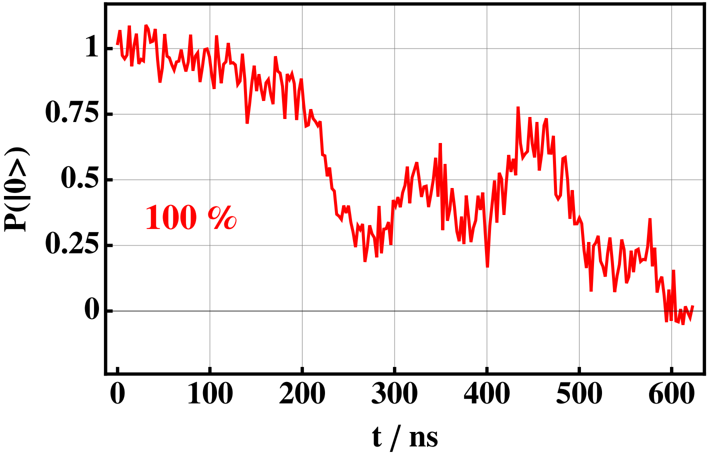
Generating π-pulses for the experiment

$\Delta\omega$: 8 MHz Gaussian FWHM, 25 % of inhomogeneity,
$n = 15$ frequency components, control amplitude < 1.5 MHz

$$\langle F \rangle > 99\,\%$$



First experimental data



experiments by Tobias Nöbauer from Atominstitut Vienna

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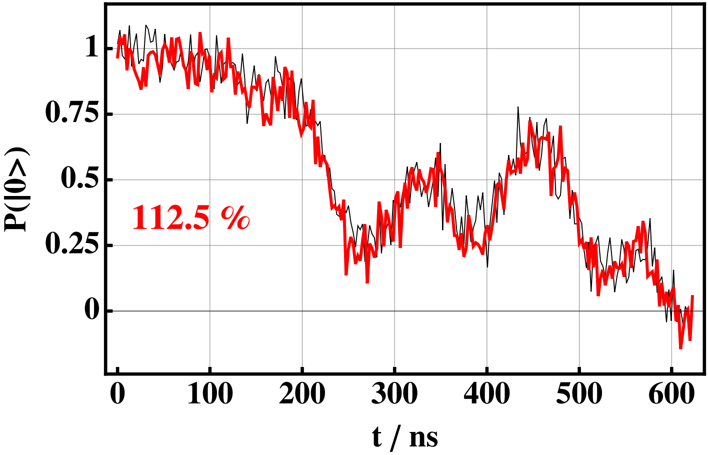

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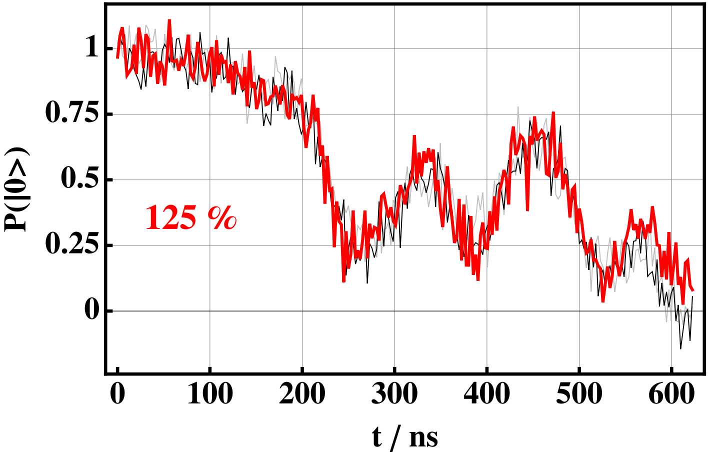

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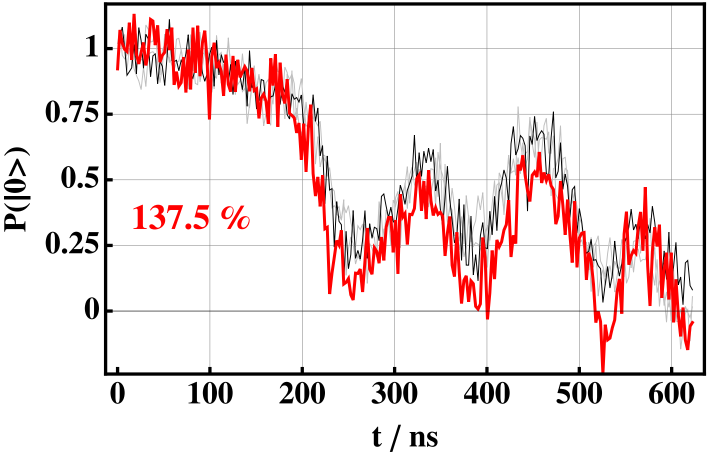

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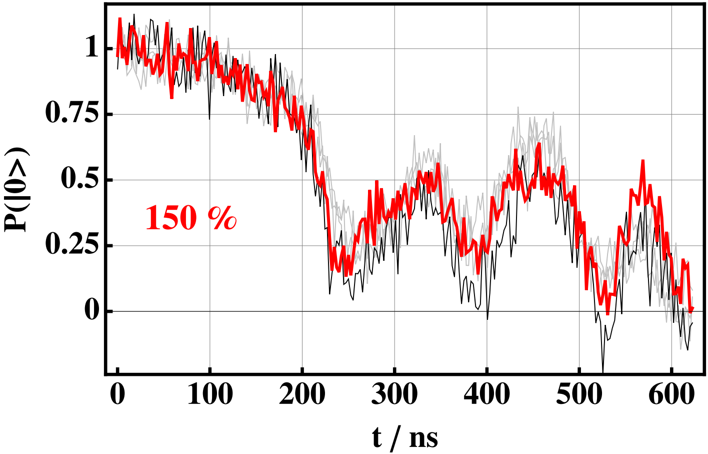

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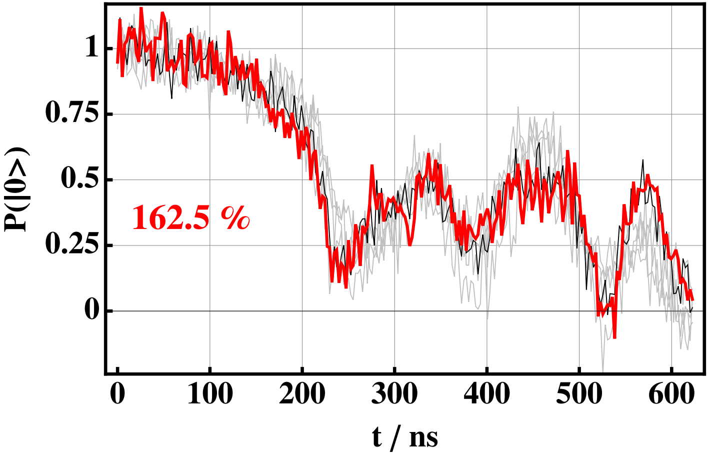

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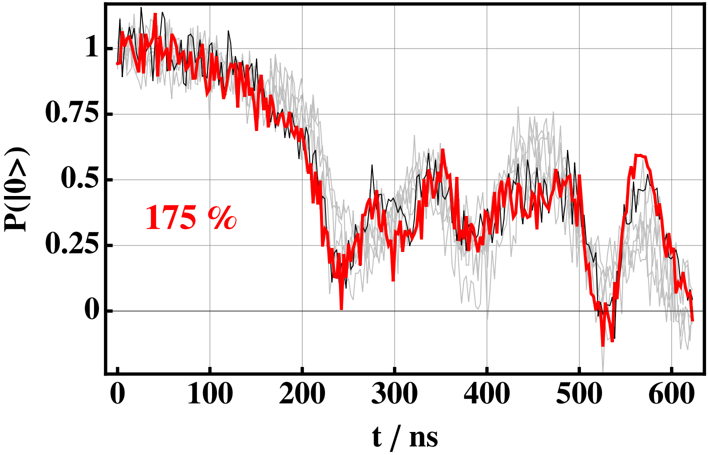

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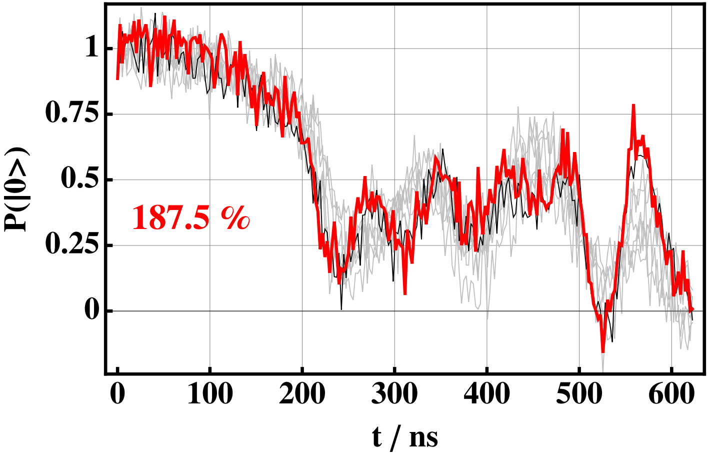

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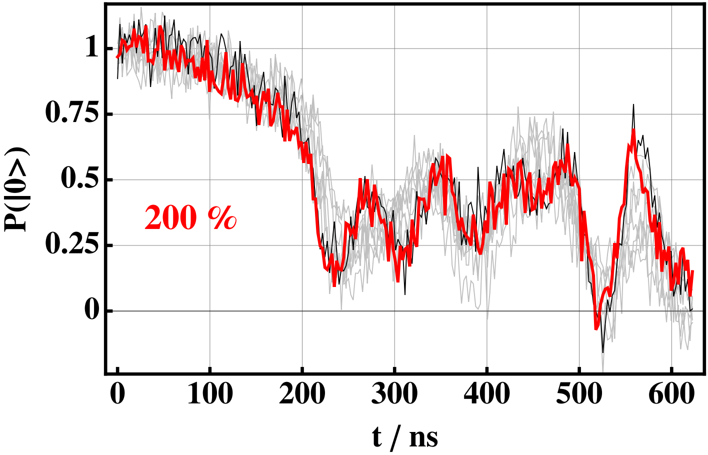

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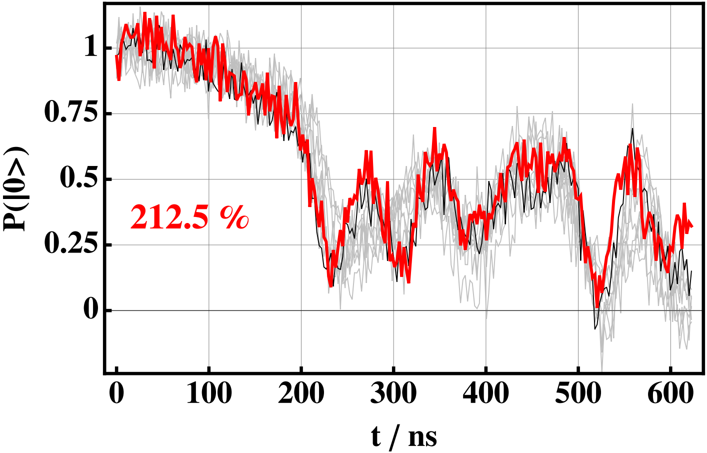

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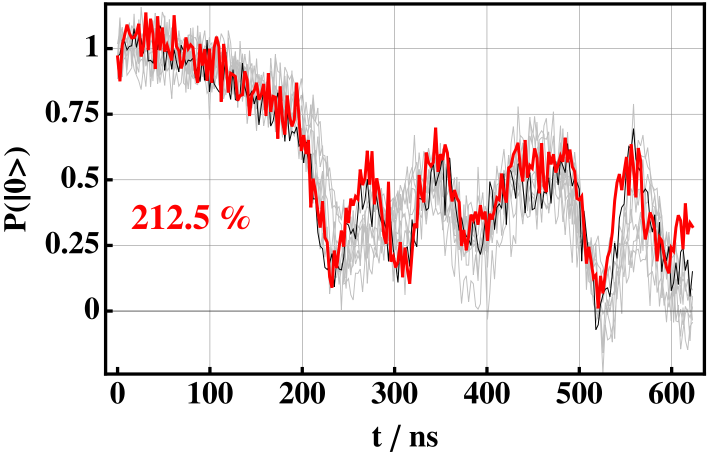

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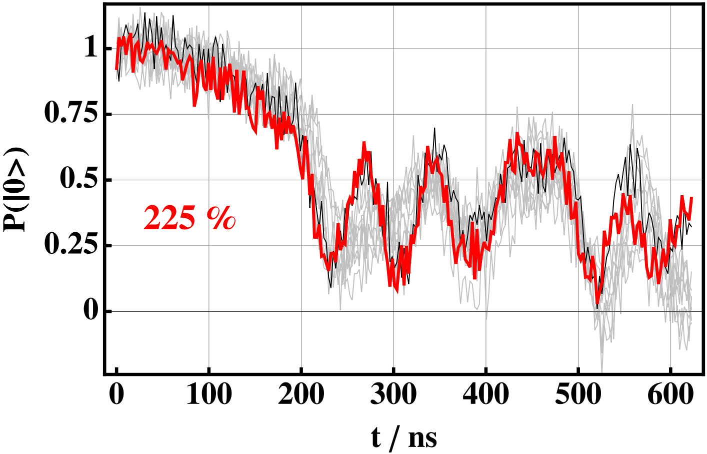

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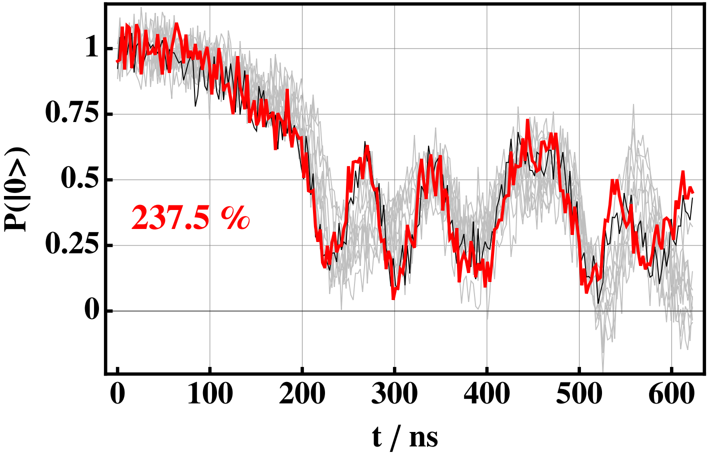

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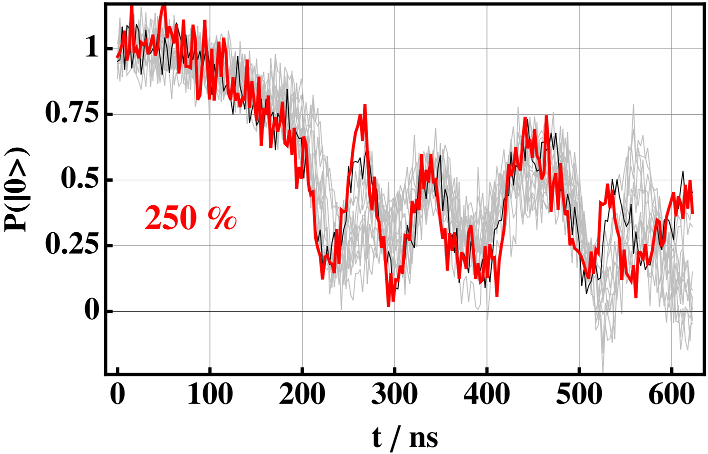

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Conclusion

Smooth optimal control

- can manipulate spin ensembles with inhomogeneous control field
- uses very simple pulses
- versatile tool that can be used with any other Hamiltonian/target functional