Optimal control with targets optimized on the fly

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Coherent dynamics in many-body quantum systems

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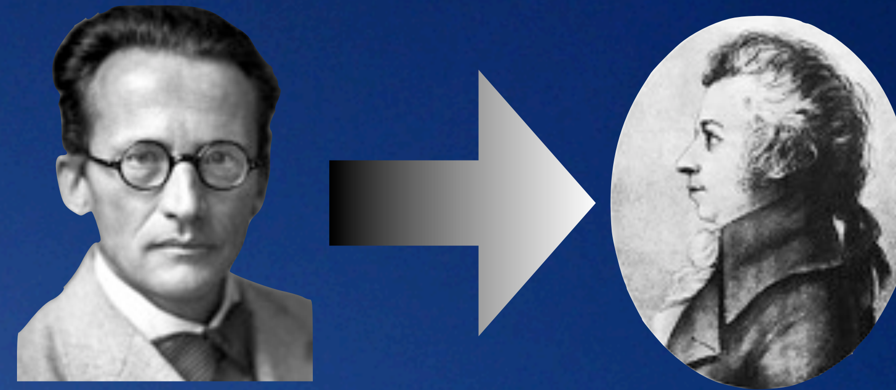
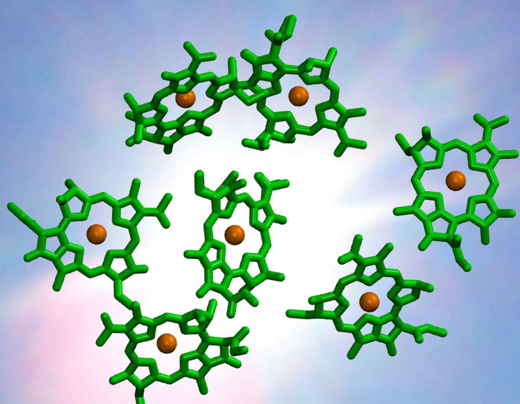
Optimal dynamical control of quantum entanglement

Why entanglement control ?

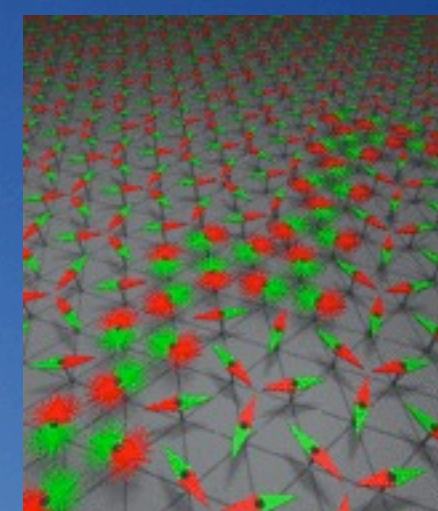
quantum-to-classical transition



identify states with robust coherence properties



identify maximally achievable range of coherence in many-body systems



Entanglement as target

Typical : define target state and maximize fidelity

continuum of equally entangled states ... $|\Phi\rangle = \mathcal{U}_1 \otimes \mathcal{U}_2 \ldots \mathcal{U}_n |\Psi\rangle$

... with different dynamical properties !

practicability

accuracy

observable
entanglement measures

$$\tau(\varrho) = \mathrm{Tr}\, \varrho \otimes \varrho\, A$$

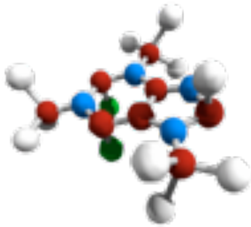
convex roof

$$E(\varrho) = \inf \sum_i p_i E(\Psi_i)$$

L. Aolita and F.M., PRL 97, 050501 (2006)
F.M. and A. Buchleitner, PRL 98, 140505 (2007)
Felix Platzer, FM, A. Buchleitner, PRL 105, 020501 (2010)

Local control

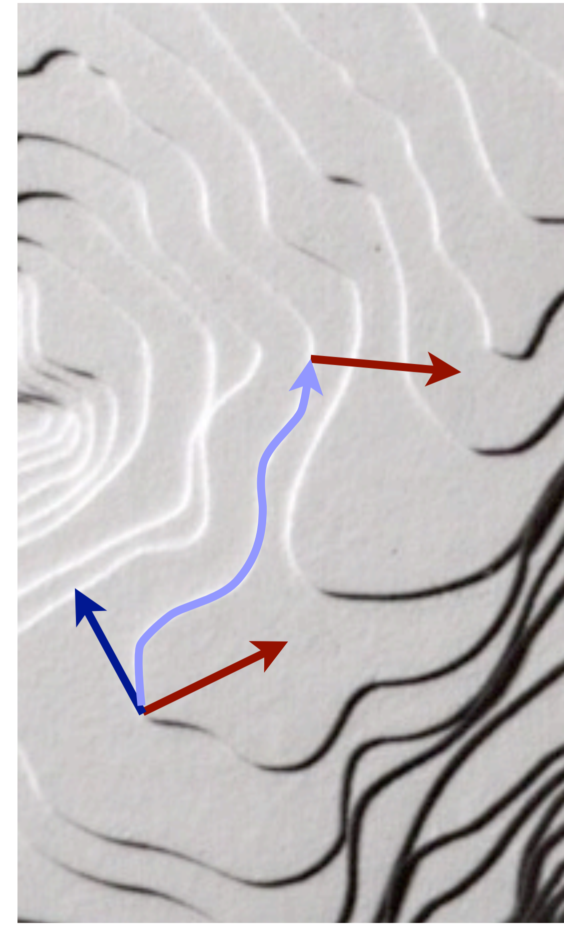
NV centers



NMR

Intrinsic interaction $H = \sum_{i \neq j} \lambda_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$

Local control $H_c = \sum_{ij} h_{ij}(t) \sigma_j^{(i)}$



$\frac{\partial \tau}{\partial t}$ entanglement is independent of local unitary dynamics

independent of control pulse

$\frac{\partial^2 \tau}{\partial t^2}$ interplay of interaction/decoherence and control

$$\frac{\partial^2 \tau}{\partial t^2} = \vec{X} \vec{H}_c + \ddot{\tau}_0$$

Entanglement as target

practicability ← → accuracy

observable
entanglement measures

reclaim invariance !

convex roof

$$\langle\Phi|\varrho|\Phi\rangle \longrightarrow \max_{\mathcal{U}_l}\langle\Phi|\mathcal{U}_l^\dagger\,\varrho\,\mathcal{U}_l|\Phi\rangle$$

$$\mathrm{tr}\,\varrho W \longrightarrow \max_{\mathcal{U}_l}\,\mathrm{tr}\;\varrho\,\mathcal{U}_l^\dagger\,W\,\mathcal{U}_l$$

$$\sqrt{\langle\Phi|\varrho^{\otimes 2}\mathbf{\Pi}|\Phi\rangle}-\sum_i p_i\sqrt{\langle\Phi|\mathcal{P}_i^\dagger\,\varrho^{\otimes 2}\mathcal{P}_i|\Phi\rangle}$$

Local control

$$\dot{\varrho} = i[\varrho, H_s + H_c] + \mathcal{L}(\varrho)$$

$$\tau(\varrho) = \mathrm{Tr}\,\varrho \otimes \varrho\, A \qquad \frac{\partial \tau(\varrho, \dot{\varrho})}{\partial t} = 2\mathrm{Tr}\,\varrho \otimes \dot{\varrho}\, A \qquad \frac{\partial^2 \tau(\varrho, \dot{\varrho}, \ddot{\varrho})}{\partial t^2} = \ldots$$

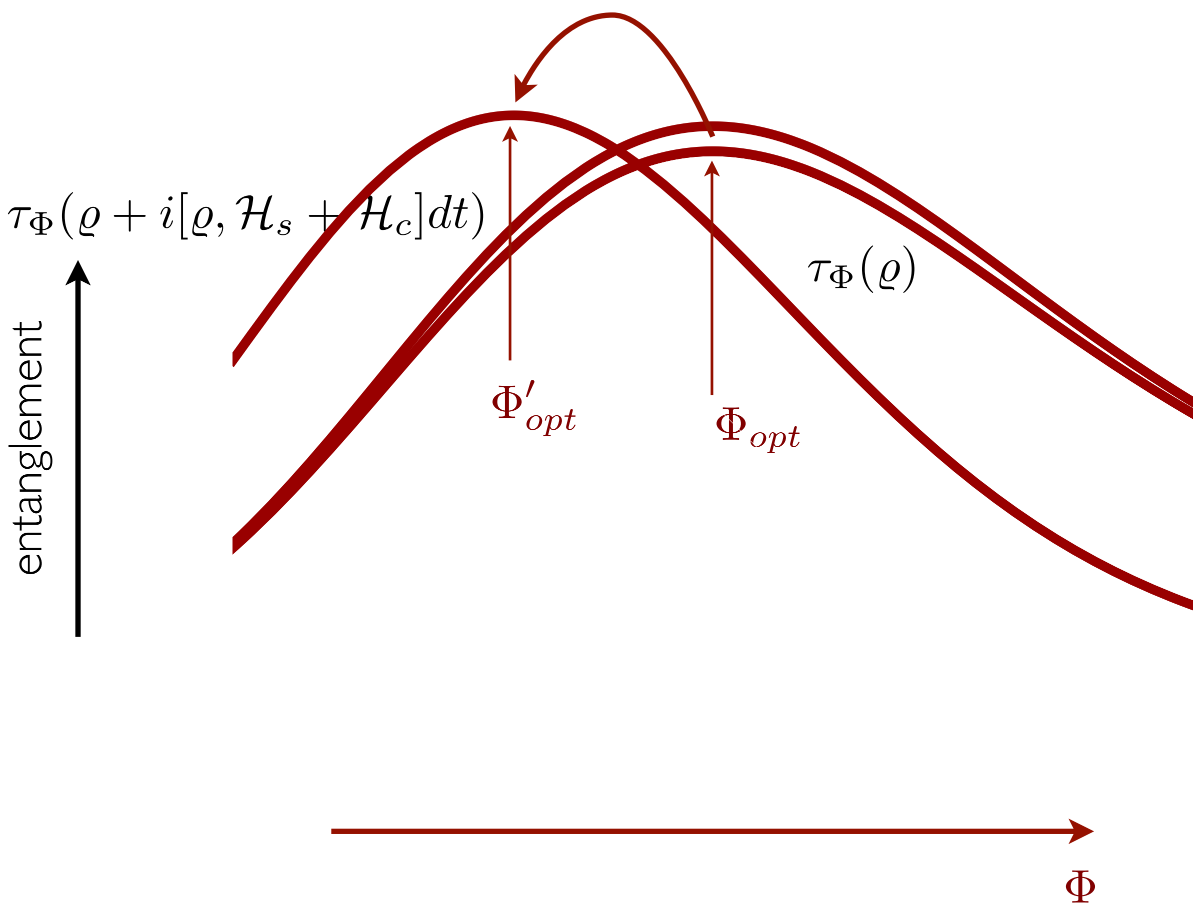
Felix Platzer, FM, A. Buchleitner, PRL **105**, 020501 (2010)

$$\tau = \max_{\Phi} \sqrt{\langle \Phi | \varrho^{\otimes 2} \mathbf{\Pi} | \Phi \rangle} - \sum_i p_i \sqrt{\langle \Phi | \mathcal{P}_i^{\dagger} \varrho^{\otimes 2} \mathcal{P}_i | \Phi \rangle}$$

$$\frac{\partial \tau}{\partial t} = \ldots \frac{\partial \varrho}{\partial t} \ldots \frac{\partial |\Phi\rangle}{\partial t} \ldots$$

to-be-solved optimization

On the fly



gradient

$$\vec{g} = \nabla_\Phi \tau$$

curvature

$$\Omega = \nabla_\Phi \nabla_\Phi \tau$$

update dummy vectors

$$\Phi \;\rightarrow\; \Phi - \Omega^{-1}\vec{g}\; dt$$

Control recipe

anticipate change of Φ-landscape for general control Hamiltonian

re-optimize τ after application of general control Hamiltonian

read off optimal control Hamiltonian ...

... pump it !

Target of control

Genuine n-body entanglement

$$\tau = \underbrace{\sqrt{\langle\Phi|\varrho^{\otimes 2}\mathbf{\Pi}|\Phi\rangle}}_{f} - \sum_i \underbrace{\sqrt{\langle\Phi|\mathcal{P}_i^\dagger \varrho^{\otimes 2}\mathcal{P}_i|\Phi\rangle}}_{f_i}$$

- τ convex
- $f, f_i \geq 0$
- $f_i(\psi) = f(\psi)$ if $|\psi\rangle$ biseparable (i-th bipartition)

M. Huber and F.Mintert, A.Gabriel and B.C.Hiesmayr PRL **104**, 210501 (2010)

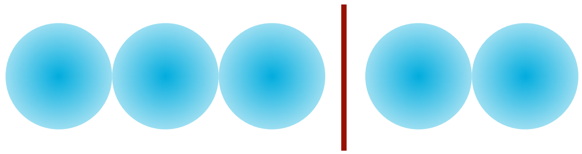
detects genuine n-body entanglement

$$\tau = f - \sum_i p_i^k \; f_i$$

tune to detect k-body entanglement

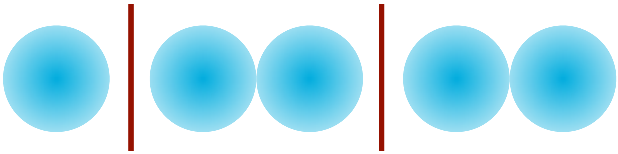
k-body entanglement

3-body entangled



separable wrt. one bipartition

2-body entangled



separable wrt. two bipartitions

$$\tau_{3,5} = f(|\psi\rangle) - \frac{1}{2}\sum_i f_i^{(2)}(|\psi\rangle)$$

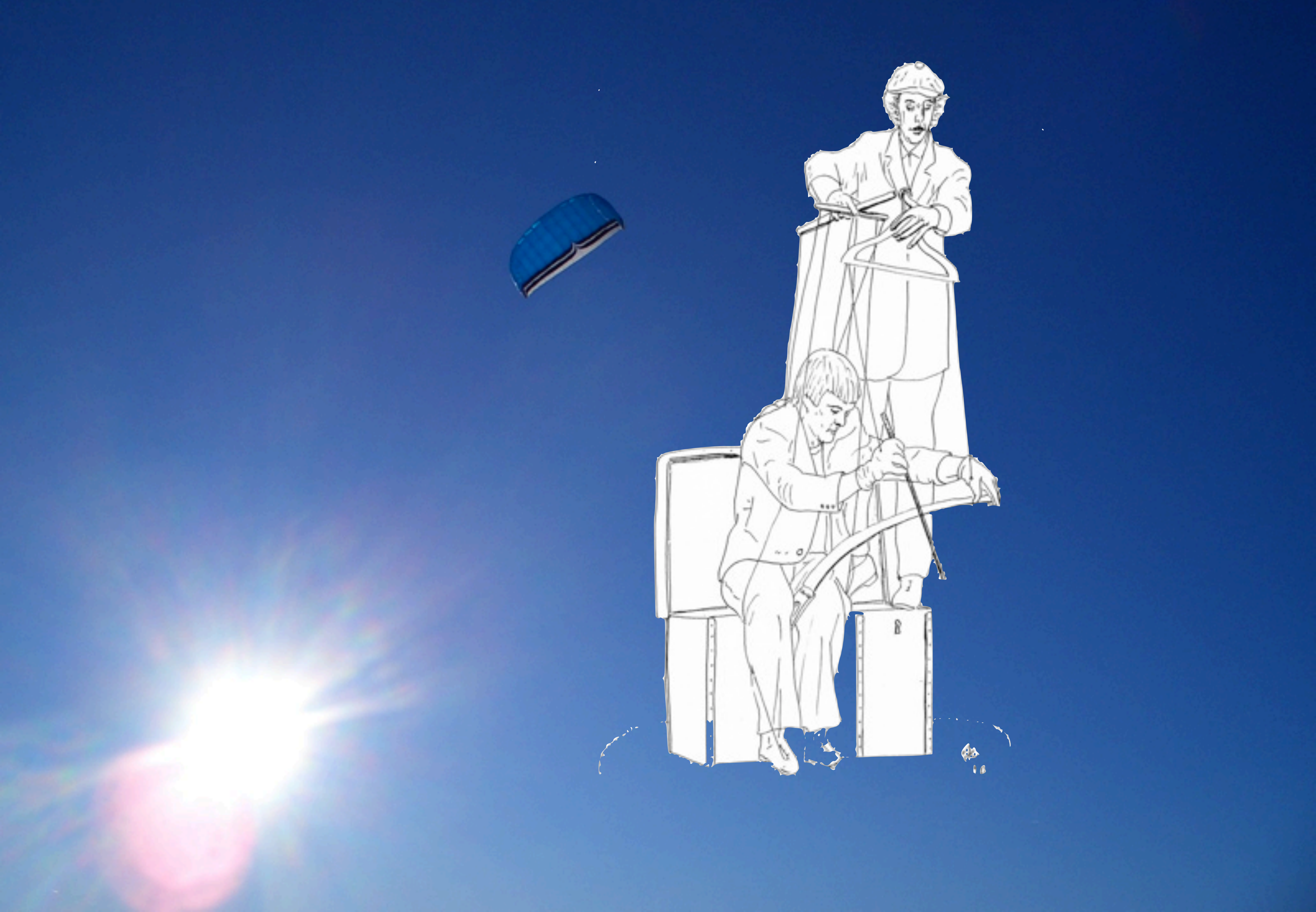
all 2-3 bipartitions

$\tau_{3,5} \leq 0$ for states without at least 3-body entanglement

$\tau_{3,5} > 0$ → at least 3-body entanglement

Federico Levi & FM arXiv:1204.5322

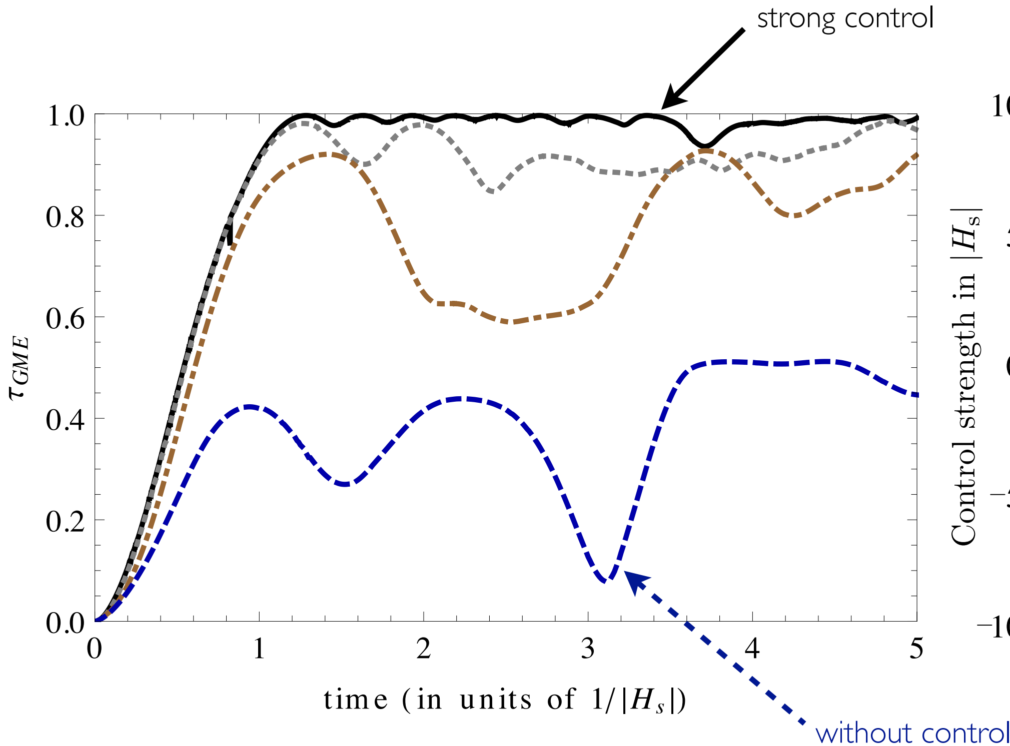
back to control

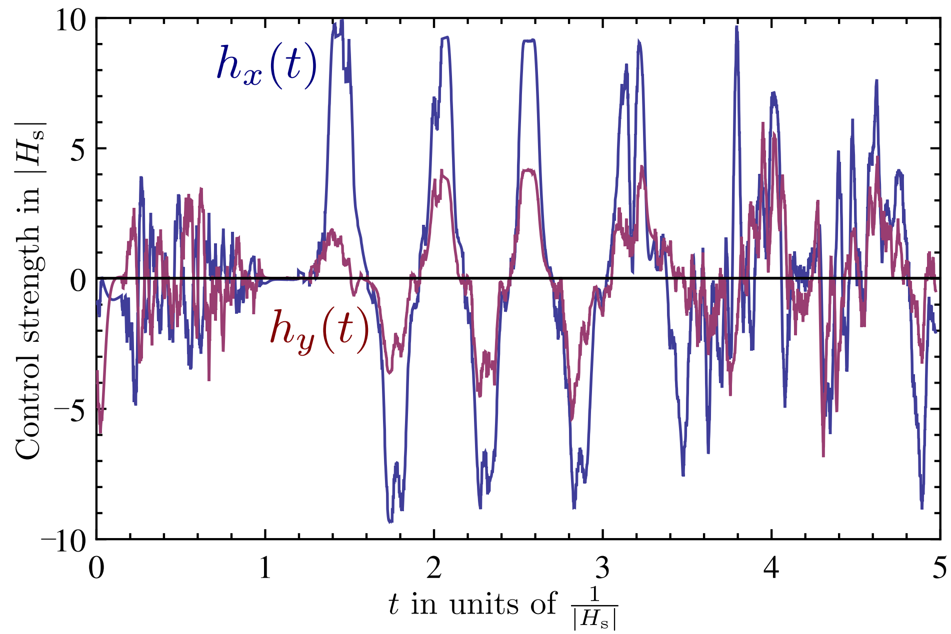


3-body system

$$\mathcal{H} = \sum_{i \neq j} \lambda_{ij} \sigma_z^{(i)} \sigma_z^{(j)}$$

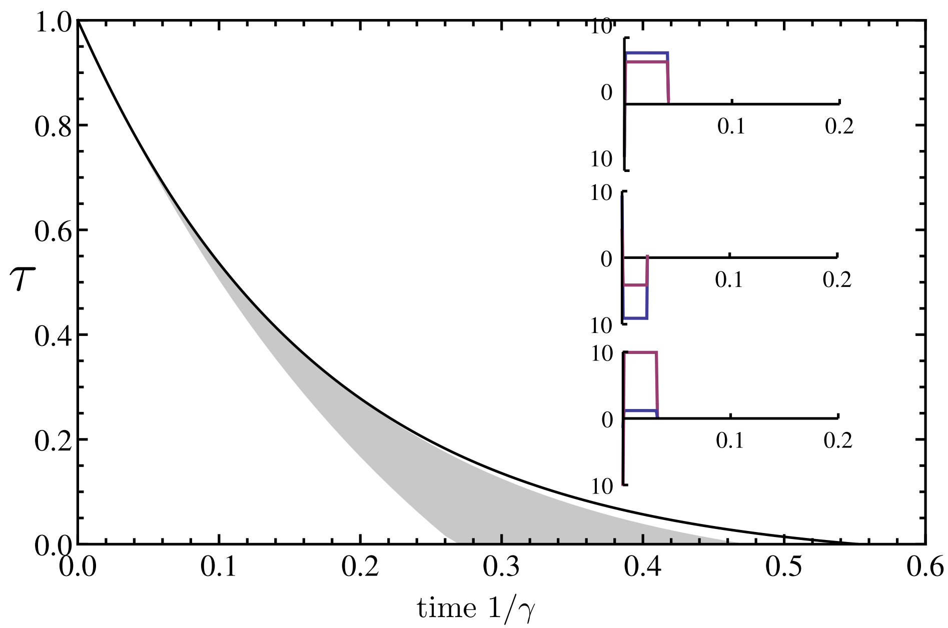
$$\mathcal{H}_c = \sum_{ij} h_{ij}(t) \sigma_j^{(i)}$$



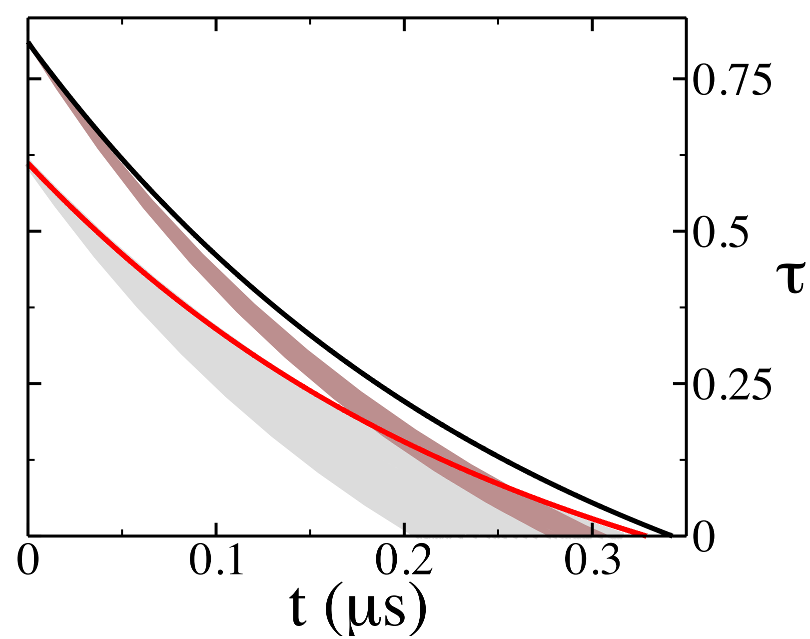


Robust coherence properties

genuine 3-body entanglement



as compared to overall entanglement



Diploma thesis Felix Platzer

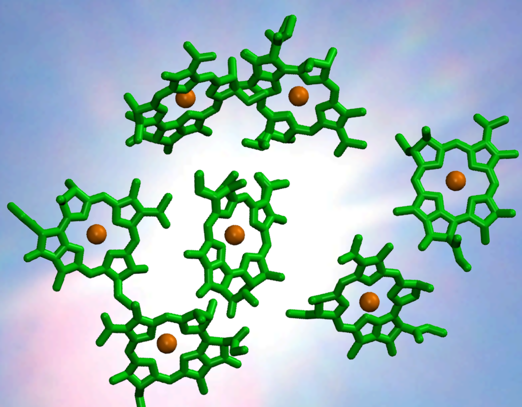
Felix Platzer, FM, A. Buchleitner, PRL **105**, 020501 (2010)

Outlook

more refined pulse-shaping

Björn Bartels on friday

identify states with robust coherence properties



engineer states with well-defined many-body coherence properties