Hard-core bosons on a triangular lattice with long range interaction with finite temperature

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Quantum Simulators

Why do we need to have quantum simulators?

Quantum Simulators

Why do we need to have quantum simulators?

- Simulating quantum mechanical systems is very difficult.
- Number of parameters that describe a quantum state grow exponentially with the number of particles. (2^n for n spin $1/2$ particles.)
- A way to solve this is to create a highly controlable system that efficiently simulates our system.

Trapped Ions

Concept

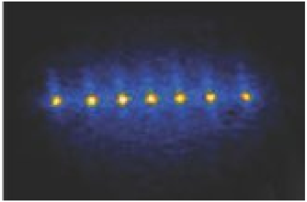
Effective Quantum Spin Systems with Trapped Ions
D. Porras and J. Cirac, *Phys. Rev. Lett.* **92**, 207901 (2004)

Proof-of-principle experiments

Simulating a quantum magnet with trapped ions
A. Friedenauer *et al.*, *Nat. Phys.* **4**, 757 (2008)

Quantum simulation of frustrated Ising spins with trapped ions
K. Kim *et al.*, *Nature* **465**, 590 (2010)

1D Spin Chain



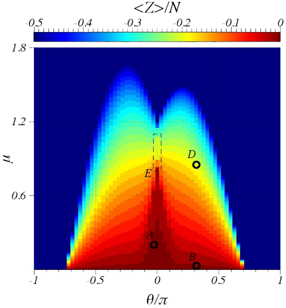
Complete devil's staircase and crystal-superfluid transitions in a dipolar XXZ spin chain: a trapped ion quantum simulation

P. Hauke *et al.*, *New Journal of Physics* **12**, 113037 (2010)

$$H = J\sum_{i,j}\frac{1}{|i-j|^3}[\cos\theta(S_i^z S_j^z) + \sin\theta(S_i^x S_j^x + S_i^y S_j^y)] - \mu\sum_i S_i^z$$

Magnetization

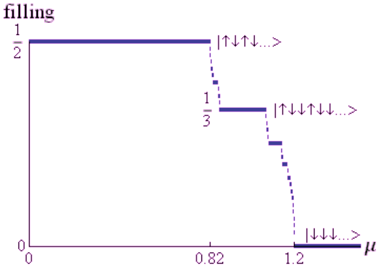
Magnetic lobes of 1D spin chain



Solved using Density Method Renormalization Group (DMRG)

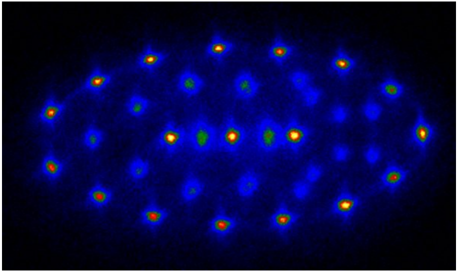
- 60 site spin chain
- Long ranged interactions
- $T = 0$
- Open Boundary Conditions.

Devil's staircase

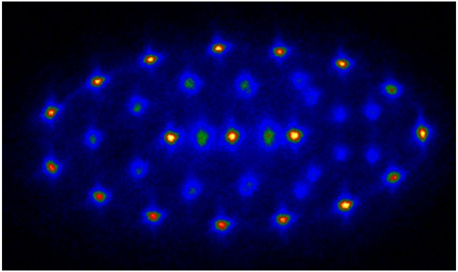


- $\theta = 0$
- Corresponds to the Ising model
- Creates a generalized Wigner crystal

The 2D model:

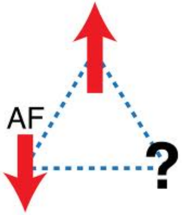


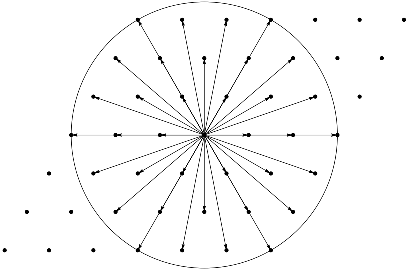
The 2D model:



- 6x6 triangular lattice with periodic boundary conditions.
- Long ranged spin-spin interactions (both hopping and dipolar)
- Ultra-frustrated

Frustration





- Prevents simultaneous minimization of interaction energies
- Creates degeneracies and a multitude of meta stable states

- NN model has 6 interactions
- LR model has 36 interactions

Holstein-Primakoff bosons

We start with our XXZ Spin Hamiltonian

$$H = J \sum_{i,j} \frac{1}{|i-j|^3} [\cos\theta(S_i^z S_j^z) + \sin\theta(S_i^x S_j^x + S_i^y S_j^y)] - \mu \sum_i S_i^z$$

Now we will use Holstein-Primakoff transformations in order to redefine our spins

$$S^- = (\sqrt{2S-n})a, \; S^+ = a^\dagger(\sqrt{2S-n}), \; S^z = n - S$$

where $n = a^\dagger a$ and $[a, a^\dagger] = 1$ and S is the total spin and the spins continue to obey their commutation relationships

$$[S^\alpha, S^\beta] = \imath\epsilon^{\alpha\beta\gamma} S^\gamma$$

Approximation

Let's take a look at the square root term.

$$\sqrt{2S-n} = \sqrt{2S}\left(1-\frac{n}{2S}\right)^{1/2}$$

Now let's expand the using Taylor series expansion

$$\sqrt{1-x} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(1-2n)(n!)^2(4^n)} x^n = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots$$

So

$$\sqrt{2S-n} = \sqrt{2S}\left(1 - \frac{n}{4S} - \frac{n^2}{32S^2} - \ldots\right)$$

Now we choose our spin to be $S = \frac{1}{2}$, then

$$S^- = a, S^+ = a^\dagger, S^z = n - \frac{1}{2}$$

Let's now apply the transformations to the Hamiltonian

$$S_i^- \rightarrow a_i, S_i^+ \rightarrow a_i^\dagger, S_i^z \rightarrow n - \tfrac{1}{2}$$

The new Hamiltonian now becomes:

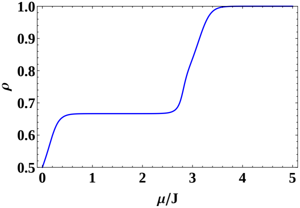
$$\begin{aligned} H &= J \sum_{i,j} \frac{1}{|i-j|^3} \left[\cos\theta \left(n_i n_j - \frac{n_i}{2} - \frac{n_j}{2} + \frac{1}{4} \right) \right] \\ &+ J \sum_{i,j} \frac{1}{|i-j|^3} \left[\frac{\sin\theta}{2} \left(a_i^\dagger a_j + a_j^\dagger a_i \right) \right] \\ &- \mu \sum_i \left(n_i - \frac{1}{2} \right) \end{aligned}$$

The simulation

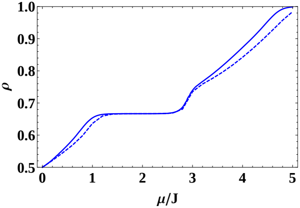
- All simulations were run using the worm algorithm of the open source ALPS (Algorithms and Libraries for Physics Simulations) project.
- This algorithm, first created by N. Prokof'ev, works by sampling world lines in the path integral representation of the partition function in the grand canonical ensemble.
- Calulations are run in low but finite temperature.
- We are restricted to only studying negative θ due to the sign problem.
- The sign problem occurs when the hopping term is negative because negative probabilities arise in the partion function.

Finite Temperature Devil's Staircase

Short ranged interactions



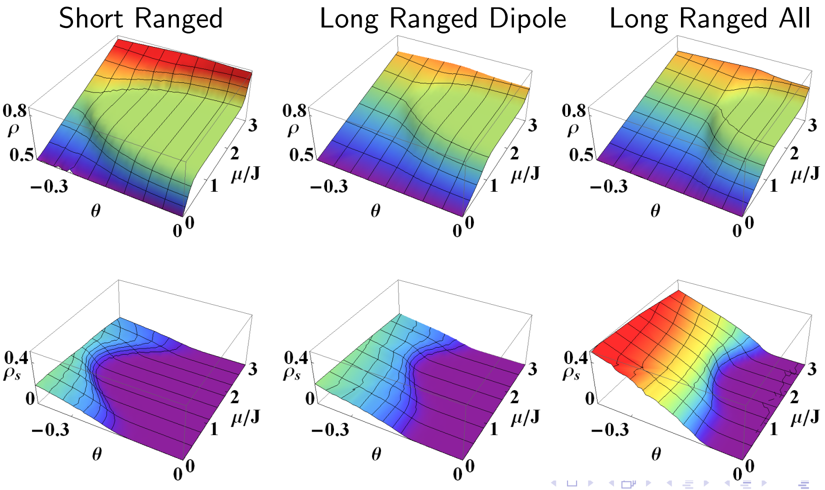
Longed ranged interactions



Wigner crystal

- $\theta = 0$
- $T = 0.1$
- 2/3 filling has largest plataeu.

Density and Superfluidity



Supersolids

In order to properly investigate the existance of a supersolid we look at the two values:

Structure factor

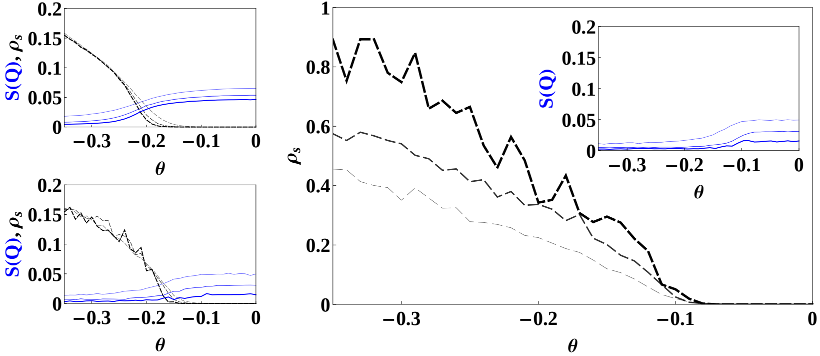
$$S(\mathbf{Q}) = \left\langle \left| \sum_{i=1}^{N} n_i e^{\imath \mathbf{Q} \mathbf{r}_i} \right|^2 \right\rangle / N^2$$

where the wave vector is $\mathbf{Q} = (4\pi/3, 0)$

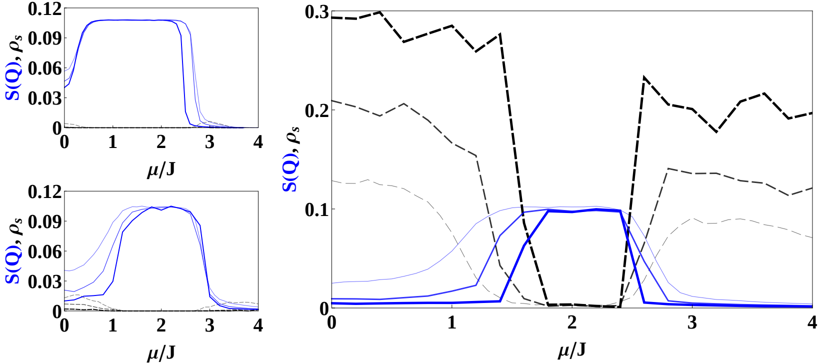
Superfluid fraction

$$\rho_s = \frac{\langle W^2 \rangle}{4\beta}$$

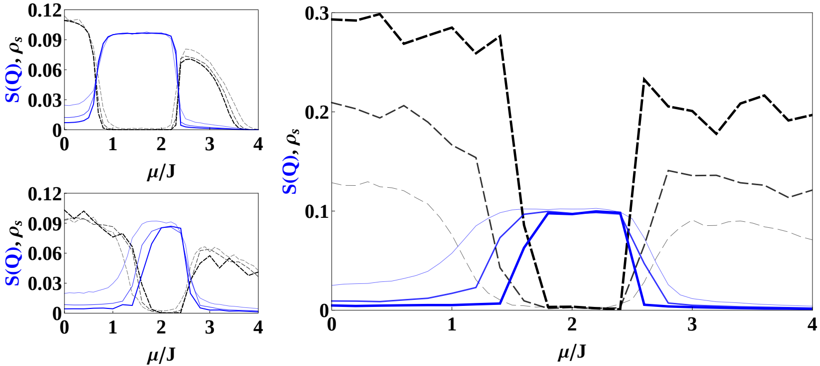
where W is the winding number fluctuation of world lines and β is the inverse temperature.



Superfluid fraction and structure factor graphs taken at $\mu/J = 0$ for multiple system sizes (L = 6, 9 and 12). Lines get thicker and darker with system size increase.



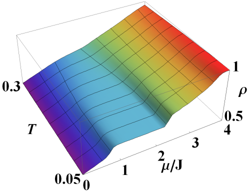
Superfluid fraction and structure factor graphs taken at $\theta = -0.15$ for multiple system sizes (L = 6, 9 and 12). Lines get thicker and darker with system size increase.



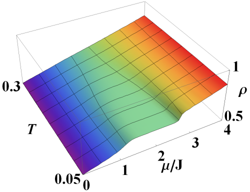
Superfluid fraction and structure factor graphs taken at 80% of the lobe ($\theta = -0.28, \theta = -0.23, \theta = -0.15$) for multiple system sizes (L = 6, 9 and 12). Lines get thicker and darker with system size increase.

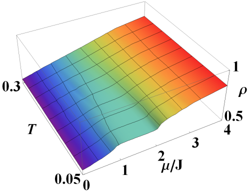
Melting of crystal lobe

Short ranged interactions



Long ranged interactions

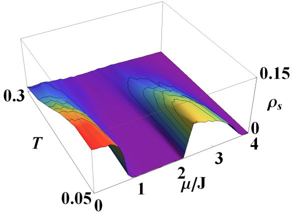


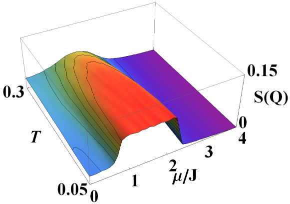


Long ranged dipolar interactions

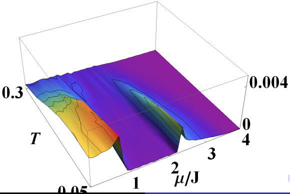
Temperature Scaling

Superfluid density and structure factor (short ranged interactions)



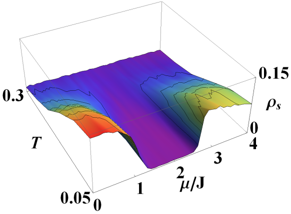


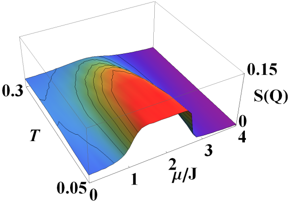
Melting of supersolid region



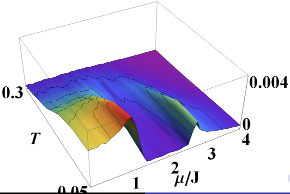
Temperature Scaling

Superfluid density and structure factor (long ranged interactions)



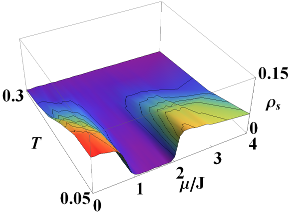


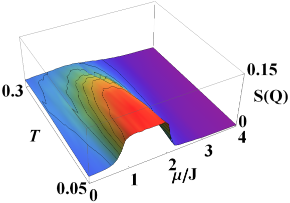
Melting of supersolid region



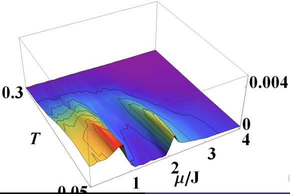
Temperature Scaling

Superfluid density and structure factor (LR dipolar interactions)



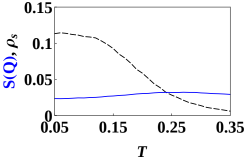


Melting of supersolid region

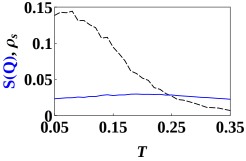


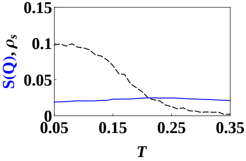
Temperature Scaling

Short ranged interactions



Long ranged interactions





Long ranged dipolar interactions

Conclusions

- Ions are a good choice for quantum simulators because of the precise control over the experimental parameters.
- Long ranged interactions reduce the size of the $2/3$ filling crystal lobe.
- Long ranged interactions stabilize the supersolid region but due to increased interactions this region melts more quickly with increased temperature.

Further Reading

Quantum spin models with long-range interactions and tunnelings: A quantum Monte Carlo study.

M. Maik, P. Hauke, O. Dutta, J. Zakrzewski and M. Lewenstein, arXiv:1206.1752 (2012)